1. Introduction

Preferential flow of water in soils, which has been widely observed in both field and laboratory studies, has effects on different processes in the vadose zone. For example, it may lead to the rapid transport of harmful chemicals to underlying groundwater resources. The resulting lack of interchange between the composite soil and the preferential flow pathways can also reduce the amount of nutrients available to plants and the amount of water in the root zone.

Preferential flow is associated with processes of macropore flow, funnel flow, heterogeneity-driven flow, and gravity-driven unstable flow. Macropore flow processes generally occur in soils with a silt or clay texture and relate to flow in non-capillary pores (Beven and Germann, 1982). Funnel flow occurs in sandy soils where inclining layers, behaving as capillary barriers, direct water into channels of concentrated flow (Kung, 1990). Heterogeneous flow occurs in soils where the heterogeneity has a strong spatial correlation (Roth, 1995). Gravity-driven fingers in unsaturated wettable porous media have been observed and analyzed on laboratory and field scales (e.g., Peck, 1965; Hill and Parlange, 1972; Raats, 1973; White et al., 1976; Glass et al., 1989; Selker et al., 1992a, 1992b; Liu et al., 1994; Lu et al., 1994; Wang et al., 1998a, 1998b; Geiger and Durnford, 2000; Yao and Hendrickx, 1996, 2001; DiCarlo, 2004). Unstable flow and fingering have also been observed in water-repellent soils which show some resistance to wetting (Hendrickx et al., 1993; Dekker and Ritsema, 1996a, 1996b, 2000; Ritsema and Dekker, 1996; Ritsema et al., 1993, 1997a,b, 1998a,b; Bauters et al., 1998; Wang et al., 2000; Wallach and Jortzick, 2008). Water-repellent soils, defined as a situation in which the soil does not wet spontaneously when water is applied to its surface, are being increasingly recognized worldwide (Wallis and Horne, 1992; Doerr et al., 2002). Despite the resistance to wetting, which is the classic demonstration of water repellency but does not reflect its full complexity, water repellency induces fingered flow. This type of flow can shorten the water infiltration time as the local wetting front in the water-repellent soil moves faster than the uniform wetting front in the wettable soil. This infiltration behavior has been observed in the laboratory (Bauters et al., 1998) and in the field (Hendrickx et al., 1993). Measurements of finger development in both wettable and water-repellent soils have revealed a decrease in moisture content and matric potential behind the wetting front (Bauters et al., 1998, 2000; Geiger and Durnford, 2000; DiCarlo, 2004; Shiosawa and Fujimaki, 2004; Wallach and Jortzick, 2008), often called saturation overshoot. It has been argued that saturation overshoot is a necessary condition.
for the production of gravity-driven fingering (Geiger and Durnford, 2000; Eliassi and Glass, 2001; DiCarlo, 2004).

In fact, water-repellent soils may be more prevalent than is commonly acknowledged (see Dekker et al., 2005 for a detailed review). Water repellency has been observed in sand, loam, clay, peat and volcanic ash soils at many locations (DeBano, 2000; Jaramillo et al., 2000). Soil water repellency, or hydrophobicity, can be particularly effective at preventing or hindering downward water movement, and creating an unstable irregular wetting front. Despite its importance and intensive research (over 1000 publications as of 2004, Dekker et al., 2005), the mechanisms leading to soil water repellency are still far from being understood, although predominant factors include soil organic matter coating of mineral grains, certain fungal, bacterial and plant species, and fire (Doerr et al., 2000). The contact angle between a water drop and a surface is a common means of characterizing whether the surface is hydrophobic (>90°) or wettable (<90°). The contact angle reaches an equilibrium value soon after its placement on surfaces with an invariant degree of repellency. In contrast, the contact angle of a water drop that is placed on the surface of a naturally induced water-repellent soil decreases gradually with time until the drop finally penetrates the soil. The time elapsed prior to water-drop penetration (WDPT) varies from a few seconds to hours and is widely used to characterize the degree of water-repellency (Lete y et al., 2000).

Many attempts have been made in the last five decades to model the unstable flow in porous media and predict the dimensions of the formed fingers (see reviews by de Rooij, 2000; Nie ber et al., 2005). As opposed to stable flow in a uniform unsaturated soil profile where flow takes place along a gradually decreasing water content gradient, sharp variations in moisture content have been observed at the contour of the unstable-flow-associated fingers. Saturation overshoot has been frequently observed at the fingers’ tips. It has been argued that the observed overshoot cannot be described by the Richards equation with standard nonmonotonic pressure saturation (P-S) and relative-permeability curves (Eli assi and Glass, 2001; Egorov et al., 2003; van Duijn et al., 2004; Fürst et al., 2009). To overcome this obstacle, Eliassi and Glass (2002, 2003) added hypodiffusive and hyperdiffusive terms to the Richards equation and Cueto-Felgueroso and Juanes (2009) included the effect of a macroscopic interface (the wetting front). As all of the models used to simulate the fingers’ dimensions and water content distribution within them require the soil’s hydraulic characteristics (retention curve, hydraulic conductivity function, sorptivity, etc.) as input, their applicability to flow in repellent soils in general, and in natural water-repellent soils (i.e. soils that render repellent due to natural processes rather than by artificial means) in particular, is under debate. This is because the soil’s characteristics and measurements of its wettability vary while the soil is in contact with water (unstable wettability), complicating the models’ implementation.

The sorptivity, S, appears in many models to predict finger dimensions in unstable flow (e.g., Par lange and Hill, 1976; Glass et al., 1990, 1991). This parameter, which is commonly used to characterize the soil’s ability to absorb water by capillary force, relates the amount of water infiltrated into a dry semi-infinite soil profile to t^{1/2} (Philip, 1957). An essential question that emerges is how the sorptivity, which is related to spontaneous wetting of soils, be determined for water-repellent soils, in which wetting requires either exertion of a positive pressure to force the water into the soil or a prolonged contact time with water prior to wetting. The sorptivity of unstable repellent soils is zero until the repellent soil becomes wettable (contact angle <90°) and infiltration begins. However, as the contact angle does not disappear all at once (if ever), the soil is considered subcritically repellent, meaning that water infiltration is not driven by capillary force that depends on the pore-size distribution per se as in fully wettable soil (contact angle ≈0). The capillary driving force at this stage is mainly controlled by the non-zero contact angle that continues to vary with time. Philip (1957) included the cosine of the contact angle in the definition of intrinsic sorptivity (which correlates the sorptivity to the dynamic viscosity and surface tension of the fluid). Philip (1971) pointed out that the dependence of intrinsic sorptivity on the contact angle is very complex, and its inclusion by multiplying the sorptivity of a “similar” wettable soil by the cosine of the contact angle is erroneous. Moreover, the role of temporally varying wettability during the contact time with water (Wang et al., 2000; Carrillo et al., 2000) in water flow in repellent soils remains poorly understood, as does the way in which this phenomenon should be combined in models that have been used to simulate unstable flow in porous media.

While the gaps in our knowledge of unstable flow in water-repellent soils exist and remain under investigation and the use of comprehensive mathematical models for water infiltration and redistribution in these soils is not feasible yet, an alternative approach is essential. The moment analysis, used to characterize spatial-temporal flow in groundwater and soils (Ye et al., 2005; Yeh et al., 2005; Lazarovitch et al., 2007), is used herein to characterize the transient shape and size of point-source induced plumes in water-repellent soils. Furthermore, we present an empirical model to relate the different plumes’ spatial moments in terms of how the flow drives the flow in soils of different water repellency degrees. The goal of this paper is to present how water flow in wettable and repellent soils behaves in terms of both physical observations and statistical measures used further to model the flow dynamics in these soils.

2. Materials and methods

2.1. Flow-chamber experiments

The experimental setup consisted of a transparent flow chamber, 32.5 cm high, 30.0 cm wide, and 0.8 cm thick (inner dimensions), packed with soils of different degrees of water repellency, and a CCD camera to record the water content distribution during wetting and subsequent redistribution processes. A high-resolution, high-speed monochromatic camera (PULNIX TM-2016-8, Sunnyvale, CA) was used to quantify light reflection from the flow chamber at a resolution of 1920 × 1080 active pixels with dimensions of 33 × 33 μm. The image frames were loaded into a PC using a frame grabber (ALACRON® Fastmotion by Alacron Inc., Nashua, NH). Two 500 W commercial halogen projectors were used to indirectly illuminate the flow chamber. A uniformly high diffusive light intensity on the chamber’s frontal face ensured maximum distinction (in terms of gray values) between dry and wetted soil. The soils used to fill the flow chamber were sampled from Safaiyeh commercial orchards located in the central region of the Israeli coastal plain. The plot from which the soil was sampled was sandy (84%, 2%, 14%, sand, silt, clay, respectively) and has been irrigated during the summer months with treated effluents for over 25 years. As a result of this prolonged irrigation with effluents, the soil’s surface layer (0–5 cm) has become water-repellent (Wallach et al., 2005). Although the soils for the current study was is the same as in Wallach and Jortzick (2008), the sampling took place at different times and locations that ensures that different soils samples have been used in both studies. The soils were sieved through a 0.5-mm sieve prior to packing. The wettable soil was taken from the road between adjacent plots, which stay dry during the irrigations; slightly water-repellent and more water-repellent soils were taken from different nearby locations along the irrigation line. The flow chamber was packed by continuously
pouring the tested soil through three randomizer screens located in the chamber’s top slit while the chamber was vibrated, ensuring homogeneous distribution of the soil in the chamber. The chamber was then clamped to a wall mount so that the cell would be fixed in the same position for each experiment, and leveled vertically. The chamber was then clamped to a wall mount so that the cell would be fixed in the chamber’s top slit while the chamber was vibrated, ensuring that the calibration line emanates from the axis origins. The slope calculation was repeated for increasing values of \( k \) is a multiplier that is not necessarily an integer. \( \kappa \) then determines the number of standard deviations in the \( x \) and \( z \) directions. Eqs. (1)–(4) were used to calculate the different moments and ellipses for the data obtained from the flow-chamber runs.

### 2.2. Moment analysis method

The 2D spatial moments for a moisture plume were defined as (e.g., Yeh et al., 2005; Lazarovitch et al., 2007)

\[
M_{ij}(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \theta_{12}(x, z, t) x^i z^j dx dz
\]

where \( \theta_{12}(x, z, t) = \theta(x, z, t) - \theta_{\text{init}}(x, z, 0) \) is the spatial distribution of the water content increase at time \( t \), and \( \theta(x, z, t) \) and \( \theta_{\text{init}}(x, z, 0) \) are the spatially measured and initial soil moisture contents, respectively. Both \( i \) and \( j \) are non-negative integers related to the \( x \) and \( z \) directions, respectively. Spatial moments are not specific to soil science and are used in general applications of pattern recognition, following for example Hu (1962). The zeroth moment, \( M_{00} \), is related to the total volume of water applied to the domain. The first moments, \( M_{10} \) and \( M_{01} \), are related to the center of mass of the wetting plume, and the second moments, \( M_{20} \) and \( M_{02} \), pertain to the amount of spreading around the plume’s mean position.

The center of mass of the plume is located at \( x_c \) and \( z_c \), given by

\[
x_c = \frac{M_{10}}{M_{00}}, \quad z_c = \frac{M_{01}}{M_{00}}
\]

The spread of the plume around its center is described by the spatial variance in the \( x \) and \( z \) directions (\( \sigma_x \) and \( \sigma_z \), respectively) expressed as

\[
\sigma_x^2 = \frac{M_{20}}{M_{00}} - x_c^2, \quad \sigma_z^2 = \frac{M_{02}}{M_{00}} - z_c^2
\]

Following Lazarovitch et al. (2007), ellipses can be defined around the center of mass \( (x_c, z_c) \) in the 2D plane. Each ellipse is represented by

\[
\frac{(x - x_c)^2}{k^2 \sigma_x^2} + \frac{(z - z_c)^2}{k^2 \sigma_z^2} = 1
\]

where \( k \) is a multiplier that is not necessarily an integer. \( \kappa \) then determines the number of standard deviations in the \( x \) and \( z \) directions. Eqs. (1)–(4) were used to calculate the different moments and ellipses for the data obtained from the flow-chamber runs.

The amount of water within each ellipse was calculated from the flow-chamber images. The fraction of the total water applied within any ellipse was calculated as the ratio between the water mass within the ellipse and the total water applied. This calculation was repeated for increasing values of \( k \), providing the cumulative probability function \( P \) from \( P = 0 \) for \( k = 0 \) to \( P = 1 \) for large \( k \) when all of the added water is included in a large ellipse (Lazarovitch et al., 2007).

### Table 1

<table>
<thead>
<tr>
<th>Soil type</th>
<th>WDPT (s)</th>
<th>Flow rate (ml/min)</th>
<th>Bulk density (g/cm³)</th>
<th>Infiltration (min)</th>
<th>Redistribution (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wettable</td>
<td>0</td>
<td>1.0</td>
<td>1.42</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3.0</td>
<td>1.43</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.0</td>
<td>1.42</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>Slightly repellent</td>
<td>23</td>
<td>1.0</td>
<td>1.33</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>3.0</td>
<td>1.31</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>5.0</td>
<td>1.32</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Strongly repellent</td>
<td>68</td>
<td>1.0</td>
<td>1.33</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>69</td>
<td>3.0</td>
<td>1.31</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Wettable</th>
<th>Slightly repellent</th>
<th>Strongly repellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate (ml/min)</td>
<td>1.0</td>
<td>3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Slope</td>
<td>0.00434</td>
<td>0.00530</td>
<td>0.00512</td>
</tr>
<tr>
<td>( r^2 )</td>
<td>0.9983</td>
<td>0.9994</td>
<td>0.9988</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.5552</td>
<td>0.9275</td>
<td>0.3042</td>
</tr>
</tbody>
</table>


Author's personal copy
3. Results

3.1. Spatial water content distribution: wetting phase

Snapshots of wetting-front propagation and spatial moisture distribution are shown in Figs. 1–3. The images of the runs with high water-application rate for the strongly repellent soil are not shown in Fig. 3 as the accumulating water flooded the soil surface, deviating considerably from the intended point-source boundary condition. The moisture content in each image pixel was determined by the respective calibration curve (Table 2) and the color code for different water content ranges appears below each figure. The wetting patterns and plume shapes were considerably different among the three soils (Figs. 1–3).

Plume width was affected by the soil’s degree of water repellency and water-application rate. While water penetrated instantaneously into the wettable soil, its penetration into the water-repellent soils was hindered and could be divided into three stages (Wallach and Jortzick, 2008). During the first stage, water accumulates on the soil surface prior to penetration. The pond’s

![Fig. 1. Moisture content distribution at a flow rate of 1.0 ml/min for different soils and infiltration times, respectively, (a) wettable, 30 min, (b) slightly repellent, 30 min, (c) strongly repellent, 30 min, (d) wettable, 60 min, (e) slightly repellent, 60 min, and (f) strongly repellent, 60 min. Insets are the horizontal cross profiles around each respective center of mass.](image1)

![Fig. 2. Moisture content distribution at a flow rate of 3.0 ml/min for different soils and infiltration times, respectively, (a) wettable, 10 min, (b) slightly repellent, 10 min, (c) strongly repellent, 10 min, (d) wettable, 20 min, (e) slightly repellent, 20 min, and (f) strongly repellent, 20 min. Insets are the horizontal cross profiles around each respective center of mass.](image2)
water flow rate, whose effect was mainly on the moisture content. Gravity on vertical water flow increased. Water application time controlled the water flux, and became elongated as the effect of capillary-tension gradients and dry soils on the two sides of the transition layer. The time of water application at which water first penetrated the soil surface while it does not penetrate the soil, owing to its water repellency. The second stage starts with water penetration and ends when the pond disappears. In the third stage, the water drops reaching the soil surface penetrate instantaneously. While moisture content at the wettable soil surface decreased gradually with radial distance from the point source down to the initial moisture content, moisture content in the two repellent soils and the pond’s maximum size (width and depth) prior to full water penetration increased with water-repellency degree and water-application rate (Table 3). Practically speaking, the pond, which transformed the point source into an area source prior to its disappearance, regulated the plume width in these soils. While moisture content at the wettable soil surface decreased gradually with radial distance from the point source down to the initial moisture content, moisture content in the two repellent soils was higher and more uniformly distributed in the horizontal direction underneath the soil surface and beyond, except for a thin transition layer at the plume edge where the moisture content decreased sharply toward its initial dry value (Figs. 1–3). Examination of the water content variation at the plume boundaries via image magnification revealed that the width of the transition layer decreases with depth and vanishes at the plume’s tip (not shown). The plumes in the two water-repellent soils formed finger-like flow at the low water-application rate (Fig. 1b, c, e and f), and in the strongly repellent soil, also at the intermediate flow rate (Fig. 2c and f). Visually, the plume area was considerably smaller with higher and more uniformly distributed moisture content compared to the wettable soil. The transition layer that bounds the high and relatively uniform water content core became thinner with increasing degree of water repellency and water-application rate (insets in Figs. 1–3). Examination of the water content variation at the plume boundaries via image magnification revealed that the width of the transition layer decreases with depth and vanishes at the plume’s tip (not shown).

### 3.2. Spatial water content distribution: redistribution phase

The spatial water content distributions in the three soils at two subsequent redistribution times are shown in Figs. 4–6 for the low, intermediate and high water-application rates, respectively. Notable differences in plume size and shape and in the internal water content distribution were obtained among the three soils and water-application rates. Occurring subsequent to the wetting stage, part of the difference was an outcome of the water-flow regime during that stage while the rest was associated with intrinsic processes of moisture redistribution. The plumes in the wettable soil continued to expand laterally and vertically during the moisture-redistribution stage, as water flowed from the wet core along a decreasing water content gradient (Figs. 4a, d, 5a, d and 6a, c). Although the extent of the difference was relatively minor, the plume size was smaller and overall moisture content higher with increasing water-application rate. These figures indicate that the course of plume expansion and internal moisture distribution shaped by the flow regime in the wettable soil during the wetting stage continued during the moisture-redistribution period.

Figs. 4b, c, e, f, 5b, c, e, f and 6b and d, indicate that during the course of moisture redistribution, the plumes in the repellent soils

**Table 3**

Soil surface water-ponding parameters.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Flow rate (ml/min)</th>
<th>Ponding time (s)</th>
<th>Ponding width (cm)</th>
<th>Ponding height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slightly repellent</td>
<td>1.0</td>
<td>–</td>
<td>0.1</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>12 ± 3</td>
<td>5.32 ± 0.560</td>
<td>0.53 ± 0.001</td>
</tr>
<tr>
<td>Strongly repellent</td>
<td>1.0</td>
<td>64 ± 5</td>
<td>5.32 ± 0.560</td>
<td>0.53 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>90 ± 10</td>
<td>5.32 ± 0.560</td>
<td>0.53 ± 0.001</td>
</tr>
</tbody>
</table>

![Fig. 3. Moisture content distribution at a flow rate of 5.0 ml/min for different soils and infiltration times, respectively, (a) wettable, 6 min, (b) slightly repellent, 6 min, (c) wettable, 6 min, and (d) slightly repellent, 12 min. Insets are the horizontal cross profiles around each respective center of mass.](image-url)
expanded slightly beyond their dimensions at the end of the wetting stage. In general, moisture content was depleted in the upper and lower parts of the plumes in these soils and remained relatively unchanged in the plumes' center. The proportions between the plume area in which moisture content remained high throughout the redistribution phase and the rest of the plume area, in which moisture content was depleted, were higher in the strongly repellent soil than in the slightly repellent soil for all water-application rates, and higher for the repellent soils with increasing water-application rates. Similar moisture-redistribution patterns were obtained in many additional runs performed in this study for different values of WDPT (results not shown), as well as in Wallach and Jortzick (2008). The similar water content patterns during redistribution indicate that they are not an outcome of

Fig. 4. Moisture content distribution at a flow rate of 1.0 ml/min for different soils and redistribution times, respectively, (a) wettable, 1 h, (b) slightly repellent, 1 h, (c) strongly repellent, 1 h, (d) wettable, 2 h, (e) slightly repellent, 10 h, and (f) strongly repellent, 10 h.

Fig. 5. Moisture content distribution at a flow rate of 3.0 ml/min for different soils and redistribution times, respectively, (a) wettable, 1 h, (b) slightly repellent, 1 h, (c) strongly repellent, 1 h, (d) wettable, 2 h, (e) slightly repellent, 10 h, and (f) strongly repellent, 10 h.
soil-profile heterogeneity which might have been unintentionally obtained during flow-chamber packing. The similarity appears to be associated with the flow regime occurring in the repellent soils, which seems to differ from that in the wettable soil (Wallach and Jortzick, 2008).

3.3. Moment analysis results: wetting phase

The different moments, $M_\mu$, were calculated using Eq. (1) for the series of water content images of the wettable, slightly repellent and strongly repellent soils. These moments were then used to...
calculate the centers of mass, $x_c$ and $z_c$, and the spatial variance in the $x$ and $z$ directions ($\sigma^2_x$ and $\sigma^2_z$, respectively) using Eqs. (2) and (3), respectively. Most of the observations presented in the current and the next chapter (Sections 3.3 and 3.4) are similar to those observed in previous chapters (Sections 3.1 and 3.2). While the observation in Sections 3.1 and 3.2 was qualitative it is qualitative in Sections 3.3 and 3.4. The drift of the center of mass in the horizontal and vertical directions during the wetting stage is presented in Fig. 7a–c, for the low, intermediate and high water-application rates, respectively. The center of mass in the horizontal direction, $x_c$, was approximately zero, indicating that it remained, as expected, in the center of the plume for all soil types and water-application rates. Similar results were obtained in a number of experimental repetitions (we repeated each case at least one more time and most cases three times, with results being very similar, but choose to focus here on individual realizations rather than deal with statistics that may blur the physics). In contrast, the pattern of center of mass vs. infiltration time in the vertical direction, $z_c$, varied with degree of water repellency and within each soil, with water-application rate (Fig. 7). While the $z_c$ curve showed a diminishing rate of increase for the wettable soil, it attained a linear path shortly after the start of the redistribution stage for the two soils (Fig. 7a–c). This transition period decreased as water repellency increased.

The drift in spatial variances in the horizontal and vertical directions ($\sigma^2_x$ and $\sigma^2_z$, respectively) is presented in Fig. 7d–f, for the low, intermediate and high water-application rates, respectively. The spatial variances in the horizontal, $\sigma^2_x$, and vertical, $\sigma^2_z$, directions increased with infiltration time. The horizontal spatial variance in the wettable soil during the wetting period was larger than the vertical spatial variance, which was limited in these experiments by the flow chamber width. The plume's approach to a constant width in the repellent soils is represented by a flattening of $\sigma^2_z$ while the $\sigma^2_x$ curves in these soils follow a linear increase.

The ellipses for one and two standard deviations around $z_c$ calculated by Eq. (4) for the measured plumes at the wetting stage, are illustrated in Figs. 1–3. The shapes of the ellipses vary among soil types and water-application rates. While the ellipses in the wettable soil were flattened, they were circle-like at early times for the slightly repellent soil and elongated thereafter. Except for early times (due to ponding), the ellipses in the strongly repellent soils were elongated throughout the runs with more elongation at lower water-application rates. In general, the external ellipses followed the shape of the plumes fairly closely.

3.4. Moment analysis results: redistribution phase

The center-of-mass drift in the horizontal and vertical directions during the redistribution stage is presented in Fig. 8a–c, for the low, intermediate and high water-application rates, respectively. Similar to the wetting stage (Fig. 7a–c), the center of mass barely moved in the horizontal direction, $x_c$, during the moisture redistribution. The variation in $z_c$ with redistribution time depended on both water-repellency degree and water-application rate. The rate at which $z_c$ increased with redistribution time in the wettable soil was low for the application rate of 1.0 ml/min and higher for 3.0 and 5.0 ml/min. The opposite held true in repellent soils, i.e., the rate of increase in $z_c$ was highest for the lowest water-application rate (Fig. 8a). The differences in $z_c$ variations at the lower and higher water-application rates were minor for the wettable soil and considerable for the repellent soils. Similar to its variation during the wetting stage (Fig. 7), the $z_c$ curve showed a diminishing rate of increase for wettable soil and a linear path shortly after the start of the redistribution stage for the two
repellent soils. The different shape of the $z_c(t)$ curves for the wettable and repellent soils during the wetting and redistribution stages, and especially the linear increase in the latter, may be interpreted as a stable flow regime in wettable soil and an unstable flow regime in the two repellent soils.

The variation in $\sigma_v$ and $\sigma_r$ is shown in Fig. 8d–f. The opposite relationships between water-application rate and plume expansion in the vertical direction are repeated in Fig. 8, namely, higher variation in moisture content along the vertical axis occurred at the lower-water-application rate in the water-repellent soils, and vice versa in the wettable soils.

The ellipses calculated by Eq. (4) for one and two standard deviations about $z_c$ for the redistribution stage are shown in Figs. 4–6. The shape of the ellipses varied among the soil types and water-application rates together with the changes in water content distribution, in a manner similar to that observed in the wetting stage. The horizontal axis of the ellipses in the wettable soil continued to be longer than the vertical axis during the 2-h moisture-redistribution period. However, the relationship between the axes is expected to change during longer moisture-redistribution periods, in conjunction with the increasing effect of gravity on plume shape. The ellipses in the repellent soils were elongated with a larger ratio between the vertical and horizontal axes in the soils having a higher degree of water repellency at lower water-application rates (Fig. 4). The ellipses for two standard deviations ($k = 2$) followed the plume shape fairly closely for the three soils (Figs. 4–6) and can therefore be used for further analyses.

3.5. Water content profiles at the end of application

The apparent differences among the shapes and internal water content distributions in the wettable and two repellent soils (Figs. 1–3) highlight the differences in flow regime, mainly between the wettable and two repellent soils. The water content distributions along a vertical cross section down the middle of the plumes in the different soils (Fig. 9) quantify these differences. The water volume in the soil profiles in Fig. 9 is 60 ml. The soil water content data was slightly smoothed by the Savitzky–Golay method (Savitzky and Golay, 1964) with a window breadth of 21 pixels. While the moisture content in the wettable soil plumes decreases gradually with depth along the vertical line forming a dispersed wetting front, the moisture content in the repellent soils is high from the wetting front (the tip) and upwards, and decreases at the upper part of plume (the tail) (Fig. 9). Such water content profiles have been denoted as saturation overshoot (DiCarlo, 2004). The tail length and the decrease in moisture content along it are affected by the degree of repellency and water-application rate (Fig. 9). The water imbibition into dry repellent soils is controlled by the rate-limited decrease in contact angle from its initial hydrophobic value ($> 90^\circ$) to the hydrophilic values ($\leq 90^\circ$) at which water can invade the pores. While water imbibition into the initially dry repellent soil is impeded, water flows through the already wetted soil, driven by capillary forces and without being impeded by high contact angle values, causing moisture to accumulate behind the wetting front. Since unstable flow has been associated with saturation overshoot behind the wetting front (Elliass and Glass, 2001; DiCarlo, 2004, 2010), it is assumed that the plume propagation in the current study can be considered unstable flow. Such a process has not been observed in the wettable soils.

Notable differences between wettable and water-repellent soils also occurred during the redistribution stage (Figs. 4–6). As opposed to the wettable soil in which the depleted moisture volume was supplemented to the dry soil during its invasion by the expanding plume, the water content depletion in the repellent soils took place primarily in the upper part of the plume, close to the soil surface, with minor changes in moisture content in the area occupied by the plume at the end of the wetting stage (Figs. 4–6). The plumes in the repellent soils barely expanded along the horizontal direction during the redistribution stage, despite the high gradient in moisture content at the plume’s edge, which indicates that flow in this direction is not driven by capillary forces per se.

4. Analysis and discussion

The effect of water repellency on the capillary driving force and the latter’s influence on the plume’s expansion with time could be described by the varying shape of the ellipses’ contours, and more specifically, by the varying ratio between the ellipses’ minor and major axes ($\sigma_v$ and $\sigma_r$, respectively, Eq. (4)). This ratio also indicates the relative effect of gravitational over capillary driving forces on the plumes’ shape and internal water content distribu-

![Fig. 9. Moisture-content distributions along a vertical cross section down the middle of the plumes at the end of the infiltration stage for different soils at different flow rates, (a) 1.0 ml/min, (b) 3.0 ml/min, and (c) 5.0 ml/min.](image-url)
The eccentricity, \( e \), which indicates the deviation of an ellipse from a circle:

\[
e(t) = \sqrt{1 - \frac{\sigma_z^2(t)}{\sigma_x^2(t)}} \quad 0 < e < 1
\]

could be used to quantify the relative effect of the flow’s driving forces on the varying plume properties with time. \( e(t) \) is defined to be zero when \( \sigma_x > \sigma_z \). The values of \( e(t) \), \( \sigma_x(t) \) and \( \sigma_z(t) \), which quantify the plume’s dispersion relative to the center of mass, are related to the capillary and gravitational driving forces. As the contact angle in repellent soils varies with time of contact between the soil particles and water, water adsorption in these soils cannot be related to a constant sorptivity coefficient, as they can in fully wettable soils. Hence, water flow by capillarity in these soils was described by a model that includes first-order processes and describes the gravitational driving force as a constant. A similar approach was successfully used by Wallach (2010) to predict the vertical and horizontal expansion of plumes that originated from a subsurface point source in similar wettable and repellent soils. The increase of the vertical axis of the ellipse with time is simulated by

\[
\sigma_z(t) = L_3(1 - e^{-k_3t}) + k_5t
\]

and of the horizontal axis by

\[
\sigma_x(t) = L_1(1 - e^{-k_1t}) + L_4(1 - e^{-k_4t})
\]

where \( L_i(t) \) expresses the maximum increase in \( \sigma \) by the first-order rate \( k_i (i = 1, 3, 4) \) that expresses the effect of capillary driving forces on the plume’s expansion. The coefficient \( k_5 \) expresses the effect of gravity as a driving force on the plume’s vertical expansion. A non-linear regression was performed to obtain the best fit of Eqs. (6) and (7) to the data in Fig. 7 and to additional plumes (not shown). The average values of the fitted parameters in Eqs. (6) and (7) and their standard deviations for all experimental runs are given in Table 4. High agreement was obtained between the fitted and measured data \( (R^2 > 0.98) \). The plumes measured in the wettable soils for a water-application rate of 1 ml/min retained an almost circular shape throughout the wetting period (Fig. 1). Therefore, Eq. (6) could not fit the \( \sigma_z \) values for these runs. These lines were extended beyond the measured 60 ml to simulate plume length after longer periods of water application (90 min). The capability of the lines obtained by fitting Eqs. (6) and (7) for 60-min data to predict the standard deviations at longer times was examined for the slightly repellent soil and the three water-application rates. The extended line fitted data that were measured for 90 min quite well (not shown).

The parameter values in Table 4 indicate that for each water-application rate, the first-order rate constants \( k_1 \) and \( k_3 \) decrease and the linear rate \( k_4 \) increases with increasing water repellency. The first-order rate constant \( k_4 \) affects the value of \( \sigma_x \) only during the initial wetting stage and is therefore affected by the flow in the soil surface vicinity, which is itself affected by the conditions at the boundary. Therefore, the ellipses’ (and plumes’) width is mainly determined by the value of \( k_4 \) (note the differences between the associated lengths \( L_1 \) and \( L_4 \) in Table 4). Note that higher values of the first-order parameters \( k_1 \), \( k_3 \) and \( k_4 \) signify that the capillary driving force is low and so is its effect on plume width, whereas higher \( k_2 \) values signify that the gravitational driving force is high, as is its effect on the plume’s downward expansion. Both driving forces are controlled mainly by the rate at which the contact angle decreases during contact of the soil with the imbibing water rather than by pore-size distribution per se. The combined relative effect of the capillary and gravitational driving forces is shown in Fig. 10 where the eccentricity (Eq. (5)) variation with time is shown for the different soils and water-application rates. While the eccentricity increases sharply for the unstable flow (repellent soils) and is slightly dependent on the water-application rate, it increases moderately following a time lag during which the plume is circular \( (e = 0) \) for the stable flow (wettable soil) and depends on the water-application rate. The empirical models above (Eqs. (6) and (7)) are located between mechanistic models and observed data. The lack of a mechanistic model to evaluate the sorptivity for

**Table 4**

<table>
<thead>
<tr>
<th></th>
<th>( L_1 )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( L_3 )</th>
<th>( L_4 )</th>
<th>( k_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wettable, 1.0 ml/min</td>
<td>( 1.070 \pm 0.095 )</td>
<td>( 0.090 \pm 0.009 )</td>
<td>( 0.126 \pm 0.033 )</td>
<td>( 0.483 \pm 0.123 )</td>
<td>( 1.580 \pm 1.028 )</td>
<td>( 0.019 \pm 0.022 )</td>
</tr>
<tr>
<td>Slightly repellent 1.0 ml/min</td>
<td>( 1.431 \pm 0.409 )</td>
<td>( 0.403 \pm 0.154 )</td>
<td>( 0.112 \pm 0.005 )</td>
<td>( 0.771 \pm 0.141 )</td>
<td>( 0.911 \pm 0.001 )</td>
<td>( 0.048 \pm 0.003 )</td>
</tr>
<tr>
<td>Strongly repellent 1.0 ml/min</td>
<td>( 0.795 \pm 0.223 )</td>
<td>( 0.517 \pm 0.182 )</td>
<td>( 0.172 \pm 0.030 )</td>
<td>( 0.910 \pm 0.108 )</td>
<td>( 1.371 \pm 0.297 )</td>
<td>( 0.427 \pm 0.493 )</td>
</tr>
<tr>
<td>Wettable, 3.0 ml/min</td>
<td>( 1.308 \pm 0.169 )</td>
<td>( 0.796 \pm 0.259 )</td>
<td>( 0.175 \pm 0.003 )</td>
<td>( 4.040 \pm 2.989 )</td>
<td>( 0.427 \pm 0.495 )</td>
<td>( 4.432 \pm 3.475 )</td>
</tr>
<tr>
<td>Slightly repellent 3.0 ml/min</td>
<td>( 0.795 \pm 0.223 )</td>
<td>( 0.517 \pm 0.182 )</td>
<td>( 0.172 \pm 0.030 )</td>
<td>( 0.910 \pm 0.108 )</td>
<td>( 1.371 \pm 0.297 )</td>
<td>( 0.427 \pm 0.493 )</td>
</tr>
<tr>
<td>Strongly repellent 3.0 ml/min</td>
<td>( 1.414 \pm 0.892 )</td>
<td>( 0.310 \pm 0.271 )</td>
<td>( 0.187 \pm 0.030 )</td>
<td>( 1.652 \pm 0.387 )</td>
<td>( 0.743 \pm 0.665 )</td>
<td>( 1.754 \pm 0.610 )</td>
</tr>
<tr>
<td>Wettable, 5.0 ml/min</td>
<td>( 1.079 \pm 0.095 )</td>
<td>( 0.090 \pm 0.009 )</td>
<td>( 0.126 \pm 0.033 )</td>
<td>( 0.483 \pm 0.123 )</td>
<td>( 1.580 \pm 1.028 )</td>
<td>( 0.019 \pm 0.022 )</td>
</tr>
<tr>
<td>Slightly repellent 5.0 ml/min</td>
<td>( 0.795 \pm 0.223 )</td>
<td>( 0.517 \pm 0.182 )</td>
<td>( 0.172 \pm 0.030 )</td>
<td>( 0.910 \pm 0.108 )</td>
<td>( 1.371 \pm 0.297 )</td>
<td>( 0.427 \pm 0.493 )</td>
</tr>
<tr>
<td>Strongly repellent 5.0 ml/min</td>
<td>( 1.079 \pm 0.095 )</td>
<td>( 0.090 \pm 0.009 )</td>
<td>( 0.126 \pm 0.033 )</td>
<td>( 0.483 \pm 0.123 )</td>
<td>( 1.580 \pm 1.028 )</td>
<td>( 0.019 \pm 0.022 )</td>
</tr>
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![Fig. 10. Ellipses eccentricities for the different soil types and water-application rates of (a) 1.0 ml/min, (b) 3.0 ml/min, and (c) 5.0 ml/min.](Author's personal copy)
The fact that a single beta distribution represents different wettable soil types, flow rates, and applied water volumes was already noted by Lazarovitch et al. (2007) for numerically derived data using a numerical solution of the Richards equation. While the theoretical basis of this universal probability distribution function has not yet been resolved, the fact that different soils, flow rates, times, and water-application rates all converge to exactly the same spatial function is striking. Furthermore, several points should be highlighted: (1) the data presented here are the first validation of Lazarovitch et al.’s (2007) findings; (2) this universal cumulative probability distribution function also applies to different degrees of soil repellency; (3) the universal function also applies to the redistribution stage of an infiltration event. Consequently, the spatial function above allows a highly accurate semi-empirical definition of subsurface water dynamics, which is highly desirable, especially for “non-classical” soils.

5. Summary and conclusions

Water repellency has a considerable impact on water flow patterns in the soil profile, which in turn affect water availability to plants and subsurface hydrology. In this study, the flow under imbibitions and redistribution conditions in wettable and water-repellent soils was analyzed. The considerable differences in the plumes’ shapes and internal water content distributions, the sharp decrease in moisture content at the plume’s edge, and the saturation overshoot behind the wetting front indicate that the flow in the water-repellent soils is unstable. The difficulty in determining the standard hydraulic characteristic functions for natural water-repellent soils, and the debatable adequacy of the Richards-equation-based models for unstable flow, call for an alternative approach to modeling flow in these soils with time-dependent contact angles. Spatio-temporal statistical method, moment analysis, was therefore applied to characterize the measured plumes in the water-repellent and wettable soils. The center of mass and spatial variances were determined for the measured evolving plumes. The change in standard deviations with time was fitted by a model that accounts for capillary and gravitational driving forces. The validated model enabled us to better predict plume shape beyond the measured periods (although one must always be cautious with extrapolation). Ellipses were defined around the stable and unstable plumes’ centers of mass, whose semi-axes represented a particular number of spatial variances, and these were used to characterize plume shape and internal water content distribution. The variation of the ellipses’ eccentricity with time indicated the enhanced role played by gravity over the diminished capillarity in the plume’s shape for different degrees of water repellency. The corresponding fractions of the total added water in the different ellipses were related to a probability curve. The single probability curve obtained for all measured plumes testified to the competence and advantage of the moment analysis method. Beyond its apparent advantages in describing the general pattern of water distribution around a surface point source in wettable soils, the major advantage of the moment analysis method is its ability to describe the dimensions and spatial water content distribution of the plumes generated by unstable flow in water-repellent soils during the wetting and subsequent redistribution stages.

The considerable differences in plume shape among the three soils and in the ratios between the vertical and horizontal dimensions of the plumes need to be addressed when a drip irrigation system is being designed. A continuous wetting strip along the lateral is usually desired in order to prevent poor seed germination and increase the root-zone volume. However, the greater depths reached by the wetting front in water-repellent soils for a given volume of applied water call for shorter irrigation intervals with

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**Fig. 11.** Cumulative probabilities as a function of the number of standard deviations, k, for various soils, flow rates, and infiltration and redistribution times. The symbols W, Sl, and St stand for wettable, slightly repellent, and strongly repellent soils, respectively. The first number in the key indicates flow rate (ml/min), the second number indicates infiltration/redistribution time. The symbols I and R stand for infiltration and redistribution, respectively. Beta is the fitted beta function (see text for values) and Lazarovitch et al. is the fitted beta function obtained by Lazarovitch et al. (2007).

The beta function parameters are $a = 3.55$ and $b = 4.35$. Fitting for individual experiments (unique soil, time and flow rate) results in very close values for a beta function. The corresponding fractions of the total added water in the different ellipses were related to a probability curve. The single probability curve obtained for all measured plumes testified to the competence and advantage of the moment analysis method. Beyond its apparent advantages in describing the general pattern of water distribution around a surface point source in wettable soils, the major advantage of the moment analysis method is its ability to describe the dimensions and spatial water content distribution of the plumes generated by unstable flow in water-repellent soils during the wetting and subsequent redistribution stages.

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less water in each irrigation event. The finger-like wetting pattern may enhance leaching of water and chemicals beyond depths at which root uptake is effective. Such overflow wastes water and may contaminate the vadose zone and groundwater with agrochemicals. Consequently, the distance between drippers along and among laterals, the drippers' discharge and irrigation scheduling should be adjusted to the soil's degree of water repellency. The information provided by the three parameters, $z_c$, $\sigma_c$, and $\sigma_w$ is comprehensive with regard to where the added water appears in the subsurface, during both the wetting and subsequent water-redistribution period in wettable and water-repellent soils. They allow the creation of an elliptical surface encapsulating any given fraction of the added water and can therefore be used in designing drip irrigation for different discharges and soil properties, without the need to determine the soil's characteristic functions and solve flow equations. Obviously, since the latter have not yet been developed for water-repellent soils, it appears that the spatial moment analysis and subsequent single fit by the beta distribution function is currently the only alternative.

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References


