Control of the Aero-Electric Power Station—an exciting QFT application for the 21st century

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SUMMARY

The Aero-Electric Power Station is the ultimate solar power station, utilizing the dry, hot air of Earth’s desert zones. By spraying water at the top of e.g. a 1200 m tall chimney with a diameter of 400 m, the air is cooled by evaporation and flows downwards through turbines at the bottom, generating 380 MW of net electric power. The Aero-Electric Power Station is still in the planning stage, and this paper belongs to a long series of feasibility studies.

The current ‘truth’ model of the Aero-Electric Power Station is a one-dimensional partial differential equation model. The external slowly changing weather, defined as the mean air pressures, temperatures and humidity at the top and bottom of the tower, determines the optimal operating point, i.e. the optimal water spray flow and turbine velocity that give the largest net power. The gross power produced by the turbine is partly delivered to the grid and partly to pump sea water to spray water reservoirs. The reservoirs make it possible to use the pumping power and the spray flow rate as control.

Wind changes cause significant deviations from the mean external air pressures, requiring closed loop regulation to keep the rotor velocity constant. The Aero-Electric Power Station may be modelled as an uncertain, unstable irrational transfer function, with two disturbances (external air pressure deviations at top and bottom), two control variables (turbine power and spray flow), and one output (rotor velocity), without a cascaded structure, giving rise to a robust load sharing control problem.

A robust linear feedback regulator is designed by QFT, in such a way that the load of regulation is shared between the two control inputs. A closed loop step response simulation for one operating condition, using the ‘truth’ model, demonstrates the design. Copyright © 2003 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The Aero-Electric Power Station is the ultimate solar power station, utilizing the dry, hot air of Earth’s desert zones. By spraying water at the top of e.g. a 1200 m tall chimney with a diameter of 400 m, the air is cooled by evaporation and flows downwards through turbines at the bottom,
generating 380 MW of net electric power. An artist’s view of an Aero-Electric Power Station is found in Figure 1. The Aero-Electric Power Station is still in the planning stage, and this paper belongs to a long series of feasibility studies. An overview of the principles and main design issues is found in e.g. References [1–3].

One of the major operational problems of the Aero-Electric Power Station is to avoid the so-called salt spray. The water sprayed at the top of the tower will of course be salt ocean water, in order not to waste costly and scarce fresh water. If the droplets evaporate completely, powdered salt will pass through the turbines and potentially harm the surroundings. Studies have shown that at optimal operation, the amount and size of the droplets should be such that the evaporation increases the salt concentration in the drops from 4% to about 10%. Then the remaining salty drops will be collected outside the turbines, and led back into the ocean. As an additional benefit, the humid outlet air will in part dew irrigate the surrounding areas. It is clear that to avoid salt spray, efficient feedback control might be of importance. The present study does not deal with the salt spray avoidance control problem directly, but with maintaining the operation near optimum.

The thermodynamic principle of the Aero-Electric Power Station is the different adiabatic heating of the humid and cool air inside the tower and the dry and hot air outside the tower, as displayed in Figure 2. The cool and humid air inside the tower is heavier than the outside air and flows downwards. The created pressure difference between the inside and outside air at the
bottom of the tower drives the turbines. The potential gross energy due to the cooling, $E_c (J/m^3)$ is given by

$$E_c = \int_0^{H_c} (\rho_a(x) - \rho_{at}(x))g \, dx \approx \rho_a g H_c \frac{T_{out} - T_{in}}{T_{in}}$$

(1)

where $\rho_a (kg/m^3)$ is the density of the air inside the tower, $\rho_{at} (kg/m^3)$ is the density of the air in the atmosphere, $g$ is the constant of gravity, $H_c (m)$ is the total height of the tower, and $T_{in} (K)$ and $T_{out} (K)$ are the temperatures inside and outside the tower, respectively. Note that $E_c$ is approximately proportional to the mean difference between $T_{in}$ and $T_{out}$, $T_{out} - T_{in}$, $T_{in}$ denotes the mean inside temperature. It is hence important to spray a sufficient quantity so that the air inside the tower is saturated with vapour and the temperature profile follows the wet adiabatic at all heights, as $T_{in}$ in Figure 2. With an insufficient spray discharge, complete evaporation will take place at some height, and from that height and below the $T_{in}$ profile will be parallel to the dry adiabatic of $T_{out}$ in Figure 2, and hence $T_{out} - T_{in}$ will be less than maximum.

A slow initial cooling at the top of the tower will decrease the value of $E_c$ in (1). It has been shown [3] that for net power optimal operation, surplus spray discharge is necessary, in order to ensure fast evaporation and a fast drop in temperature at the top of the tower, see Figure 2. Consequently, water drops will exit the turbines together with the humid air. Ideally, infinitesimal droplets should be used. Available spraying equipment is capable to produce droplets with a diameter of 150 $\mu$m. The optimal surplus discharge is such that the size of the droplets will decrease to a diameter of about 110 $\mu$m at exit, thus also avoiding salt spray. As stated above the salt concentration in the droplets will increase from 4% to 10%, the latter figure also being the limit for efficient water vapour evaporation from the droplets at the bottom of the tower. Larger surplus spray discharge will increase gross power production, since a large part of the kinetic energy of the remaining droplets can be retrieved in the turbines, but net power production will decrease due to the pumping losses.

The power production is proportional to the product of air mass flow, and the air pressure difference between inside and outside air at the bottom of the tower, (or, alternatively, head at the turbine). Both of these variables are functions of the entrance velocity of the air into the tower, $v_{top} (m/s)$. In Reference [4], on which this paper is based, it is shown that in steady state, $v_{top}$ is a monotonous function of the turbine rotor angular velocity, $n$ (rpm), for the studied weather conditions. It can be shown [3], that there exists an optimal constant entrance velocity for which the net power production is maximized. Figure 3 shows the potential net power as a
function of constant spray rate and constant \( v_{\text{top}} \); and the dependence of the net power on constant spray rate and constant rotor velocity.

The ‘truth’ model of the Aero-Electric Power Station is a one-dimensional partial differential equation model, having the air density, the air velocity, the temperature, the humidity of the air and the mass of the evaporating droplets as state variables, developed in Reference [5]. The ‘truth’ model took its current form in Reference [4] where Borshchevsky’s model was combined with a variable rotor speed turbine model, [6], whose state variables are the air velocity through the turbine and the rotor angular velocity. The model is described in Section 2.

The external weather conditions, defined as the air pressures, temperatures, and humidity at the top and bottom of the tower, determines the optimal operating point, i.e. the optimal water spray flow and turbine velocity that give the largest net power. The gross power produced by the turbine is partly delivered to the electric grid that is assumed to accept all it gets, and partly to pump sea water to the lower water reservoir at the bottom of the tower, and from the lower reservoir to the upper reservoir at the top. The reservoirs make it possible to use the pumping power as a control input in addition to the spray flow.

In Figure 3, the net power \( N_{\text{net}} \) (W) is displayed for combinations of the spray rate \( Q_w \) (m\(^3\)/s) (from which the pumping power \( N_p \) (W) follows) and rotor velocity \( n \) (rpm). In this way the optimal operating point is found for a given weather condition, yielding set points for all input and output variables.

For each operating condition it is possible to ‘locally’ model the Aero-Electric Power Station plant as an uncertain unstable irrational transfer function, with the deviations from the nominals of the delivered turbine power (or equivalently rotor load torque) and spray flow as

\[
\begin{align*}
\text{Figure 3. Left: Potential net power (MW) as function of constant entrance air velocity, and constant spray rate, for a 1200 m high tower with diameter of 400 m, sprayed with droplets of 150 \text{ \mu m} diameter. An ideal tower model with an ideal turbine is used for the computation. The weather conditions are } T_{\text{out(top,t)}} &= 293 \text{ (K), } T_{\text{out(bottom,t)}} = 304 \text{ (K), atmospheric vapour density (humidity) = 20 \text{ kg water/kg air), } P_{\text{out(top, t)}} = 93000 \text{ (Pa), } P_{\text{out(bottom,t)}} = 106700 \text{ (Pa), where } t \text{ denotes time (s). Right: Net power (MW) as a function of constant rotor angular velocity and constant spray discharge. The computation is done with a static model that approximates the ‘truth’ model in steady state with at most 15\% error [4], and the turbine described in Section 2.}
\end{align*}
\]
control inputs, and the deviation from the nominal of the rotor velocity as the output. Changes of external humidity, temperatures, and mean external air pressures are typically very slow with diurnal and slower variations, and hence these changes can be taken into account by slowly changing the operating condition. Wind changes, however, will cause significant disturbances in the external air pressures at the top and bottom of the tower in the frequency range 0.002–0.2 Hz, according to Reference [7]. Therefore deviations from the nominals of the external top and bottom air pressures are included as disturbances in the model for regulation, whereby the regulator is to be designed to keep the rotor velocity constant at its nominal value. Thus the plant model has two disturbances (external air pressures at top and bottom), two control variables (turbine power, and spray flow), and one output (rotor velocity), without a cascaded structure. The local transfer function model is described in Section 3.

A robust linear feedback regulator is designed by QFT in such a way that the load of regulation is shared between the two control inputs, using the load sharing ideas of Reference [8]. The design was done with Qsyn—the Toolbox for Robust Control Systems Design [9]. A closed loop simulations using the ‘truth’ model is presented in Section 5. The results and their implications are discussed in Section 6. One of the conclusions is that QFT is eminently suited to solve this challenging control problem.

2. ONE-DIMENSIONAL ‘TRUTH’ MODEL

In this section a short overview is given of the computational one-dimensional partial differential model of Reference [5] which is partially based on Reference [1]. In this model, the tower is sliced into \( h = 20 \) m tall slices (cells), and the time is discretized by \( \theta = 0.05 \) s, such that a pressure wave that travels with the speed of about 340 m/s will hit each slice at least once in discrete time. Let \( t \) denote the time (s), and \( x \) the distance from the top of the tower (m). The balance equations defining the operation inside the tower follow.

The **air mass balance equation** (continuity equation)

\[
\frac{\partial \rho_a}{\partial t} + U \frac{\partial \rho_a}{\partial x} + \rho_a \frac{\partial U}{\partial x} = S_v
\]  

(2)

where \( \rho_a \) (kg/m\(^3\)) is the humid air density, \( U \) (m/s) is the air velocity, and \( S_v \) (kg/m\(^3\)/s) is the vapour source. The **momentum balance equation** is

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \frac{1}{\rho_a} \frac{\partial P}{\partial x} = g - f \frac{U^2}{2H_c} + \frac{S_F}{\rho_a}
\]  

(3)

where \( R \) (J/kg/K) is the gas constant for air, \( T \) (K) the temperature of the air, \( g \) (m/s\(^2\)) the constant of gravity, \( f \) the friction coefficient, \( H_c \) (m) the height of the tower and \( S_F \) (N/m\(^3\)) is the momentum source due to drag. The **energy balance equation** is

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + \frac{P}{\rho_a C_v} \frac{\partial U}{\partial x} = \frac{U^3}{2C_v} f - \frac{S_Q(x,t)}{\rho_a C_v}
\]  

(4)

where \( C_v \) (J/kg/K) is the specific heat of air at constant volume, and \( S_Q \) (J/m\(^3\)/s) is the energy source caused by the interaction with the water drops. The **vapor balance equation** is

\[
\frac{\partial \omega_b}{\partial t} + U \frac{\partial \omega_b}{\partial x} + \omega_b \frac{\partial U}{\partial x} = S_v
\]  

(5)
where \( \omega_b \) (kg water/m\(^3\) air) is the vapour density. The equations for the \textit{mass} of a drop, \( m_d \) (kg), the \textit{drop diameter}, \( d_d \) (m), and the \textit{velocity} of a drop, \( U_d \) (m/s), are

\[
\frac{dm_d}{dt} = -\rho_d \sigma D_d d_d (\omega_d - \omega_a)
\]

\[
d_d = 3 \sqrt{\frac{6m_d}{\pi \rho_d}}
\]

\[
U_d(x, t) = U(x, t) + \frac{g \rho_d d_d(x, t)}{18 \mu_a (1 + 0.15R_e \omega_a^{0.687})}
\]

where \( \sigma \) is the Sherwood number, \( D_a \) is the diffusion coefficient, \( d_d \) (m) is the water drop diameter, \( \omega_d \) (kg water/kg air) is the saturation humidity of the air, \( \rho_d \) (kg/m\(^3\)) is the density of the drop, \( \mu_a \) (kg/m/s) is the elasticity of the air, \( R_e \) is the Reynolds number, and \( \omega_a \) (kg water/kg air) is the humidity of the air, computed from (5). The \textit{vapour source} in (1) and (5) is given by

\[
S_v = -\frac{6Q_w}{\pi d_d^3 A_e U_d} \frac{dm_d}{dt}
\]

where \( A_e \) (m\(^2\)) is the cross-section area of the tower. The \textit{momentum source due to drag} in (3) is

\[
S_F = \frac{6Q_w}{\pi d_d^3 A_e U_d} 3\pi d_d \mu_a (1 + 0.15R_e^{0.687}) (U_d - U)
\]

The equation for the \textit{energy source} caused by the interaction with the water drops is

\[
\frac{\partial T_d}{\partial t} = \frac{Nuk_a \pi d_d (T_d - T) + L \frac{dm_d}{dt}}{m_d C_{drop}}
\]

where \( v \) is the Nusselt number, \( k_a \) (J/(Ksm)) is the thermal conductivity of air, \( T_d \) (K) is the temperature of the drops, and \( L \) (J/kg) is the evaporation energy for water. \( C_{pd} \) (J/K/kg) is the drop specific heat at constant pressure.

The boundary conditions for the tower are governed by the following equations. The \textit{humid air density at the top of the tower}, \( \rho_a(0, t) \) (kg/m\(^3\)) is given by

\[
\rho_a(0, t) = \frac{P_{out}(top, t)}{RT_{out}(top, t)}
\]

where \( P_{out}(top, t) \) (Pa) is the outside atmospheric air pressure at the top of the tower, \( T_{out}(top, t) \) (K) is the outside atmospheric temperature at the top of the tower, and, as mentioned above, \( R \) (J/kg/K) is the gas constant for air. The \textit{air velocity at the bottom of the tower}, \( v_{bot}(t) = U(H_c, t) \) (m/s) is given by

\[
v_{bot}(t) = \frac{m \pi D^2 v_{tur}(t)}{4A_e}
\]

where \( m \) is the number of turbines, \( D \) (m) is the diameter of the turbine, and \( v_{tur}(t) \) (m/s) is the velocity of the air through the turbines, given below by (12). \( T_{out}(top, t) \) constitutes the boundary condition for (4), while the atmospheric vapour density equals \( \omega_b(0, t) \) in (5). The initial water drop velocity at the top of the tower, \( U_d(0, t) \), is neglected since it tends very quickly to the drop
velocity in (6). The initial drop diameter, \( d_{0} = d_{0}(0, t) \) (m), is assumed to be constant in this study = 150 \( \mu \)m, but could be used as a control variable. The initial mass of a drop follows from (6). The spray flow rate, \( Q_{w}(t) \) (m\(^{3}\)/s), is a control variable, follows from \( d_{0}(0, t) \) and (6).

The turbine model is adapted from Reference [6], and contains two differential equations. The differential equation for the air velocity through the turbine, \( v_{\text{tur}}(t) \) (m/s), is

\[
\dot{v}_{\text{tur}}(t) = \frac{C_{p}}{2L_{T}}(v_{\text{pot}}^{2}(t) - v_{\text{tur}}^{2}(t))
\]

where \( C_{p} \) is the efficiency of the turbine and diffuser system, given by a table [12], \( L_{T} \) (m) is the length of the turbine, and \( v_{\text{pot}}(t) \) (m/s) is the ‘potential’ air velocity, given by

\[
v_{\text{pot}} = \frac{25600(3200n(t))p + 2\sqrt{-82944q^{2}n(t)}^{2} + 81q^{2}H_{\text{pot}} + 2500p^{2}H_{\text{pot}}q^{2}}{81q^{2} + 2500p^{2}} \cdot 4 \pi D^{2}
\]

\[
H_{\text{pot}}(t) = \frac{P_{\text{in}}(H_{c}, t) - P_{\text{out}}(H_{c}, t)}{g\rho_{a}(H_{c}, t)}
\]

\[
P_{\text{in}}(H_{c}, t) = \frac{\rho_{a}(H_{c}, t)}{RT}(H_{c}, t)
\]

where \( n(t) \) (rpm) is the rotor angular velocity, and \( p \) and \( q \) are constant turbine parameters, \( H_{\text{pot}} \) (m) is the ‘potential’ head, computed according to static head in Reference [10], \( P_{\text{in}}(H_{c}, t) = P_{\text{in}}(\text{bottom}, t) \) (Pa) is the internal pressure at the bottom, \( P_{\text{out}}(H_{c}, t) = P_{\text{out}}(\text{bottom}, t) \) (Pa) is the atmospheric pressure at the bottom, see the atmospheric model in Reference [11]. \( P_{\text{in}}(0, t) = P_{\text{out}}(\text{top}, t) \) is a boundary condition. \( T(x, t) \) is given by (6), \( T(0, t) = T_{\text{out}}(\text{top}, t) \) is the boundary condition for (6), and \( R \) (J/kg/K) is the gas constant for air. Note that it is via (13), (11) that the water spray enters as a control variable that influences the air velocity through the turbine in (12). The equation for \( v_{\text{pot}} \) is valid for the flow resistance coefficient \( R_{f} = 0.0324 \), see Reference [4], Turbine Diameter of 32 (m) and 100 turbines at the tower bottom.

The differential equation for the rotor angular velocity, \( n(t) \) (rpm), is

\[
\dot{n}(t) = \frac{1}{J_{T}} \left( \frac{\pi \rho_{a} D^{2} v_{\text{pot}}^{3}(t)}{8n(t)} C_{p} - \left( \frac{2\pi}{60} \right)^{2} \frac{N_{\text{in}}(t)}{mn(t)} \right)
\]

where \( J_{T} \) (kgm\(^{2}\)) is the turbine moment of inertia, \( m \) is the number of turbines, and \( N_{\text{in}}(t) \) (W) is the power delivered from the turbines. Note that

\[
N_{\text{in}}(t) = N_{\text{net}}(t) + N_{p}(t)
\]

where \( N_{\text{net}}(t) \) (W) is the net power delivered to the grid, and \( N_{p}(t) \) (W) is the pumping power. The pumping power needed to lift the water from the bottom of the tower to the sprayers is

\[
N_{p}(t) = \frac{\rho_{w}gQ_{w}(t)(H_{c} + H_{\text{loss}})}{\eta_{p}}
\]

where \( H_{\text{loss}} \) (m) is the loss of head in the water intake and in the sprayer, and \( \eta_{p} \) is the efficiency of the pumps. When optimizing the steady state operation, (16) is used. (16) prescribes that \( N_{p}(t) \) cannot be a control variable independent of \( Q_{w}(t) \). If there is water storage capacity at the bottom and top of the tower, \( N_{p}(t) \) may however be used as an independent but limited control variable. In this paper it is assumed that for dynamic regulation, \( Q_{w}(t) \) and \( N_{p}(t) \) are independent, and that the capacities of the storage tanks are sufficient.
3. LINEAR MODEL AROUND AN OPTIMAL OPERATING POINT

With the tower-and-turbine model in Section 2 in steady state, and a given constant weather it is possible to find the constant spray flow rate \( Q_w \) and rotor angular velocity \( n \) that maximizes the net production of electrical power, \( N_{\text{net}} \), see Figure 3.

The results in Figure 3 show that the optimum is not very sensitive to \( Q_w \in [16, 18] \text{ m}^3/\text{s} \) for \( n \approx 30.7 \text{ rpm} \). An insufficient spray flow (\( Q_w < 16 \text{ m}^3/\text{s} \)) causes the net power to decrease sharply. The decrease of turbine efficiency contributes to the sharp decrease in net power for insufficient spray flow. Surplus spray flow also decreases net power but less drastically, since the potential energy of the water drops may be partly recovered. This fact is beneficial for avoiding salt spray.

In view of Figure 3, one possible way to control the aero-electric power station is to make use of an extremum-seeking controller [13] whereby set points for \( Q_w \) and \( n \) are ‘probed’ in the vicinity of the current set point, in order to track an optimum that is changing slowly due to the slowly changing weather conditions. In this paper another approach is taken. It is assumed that a pre-computed operating table prescribes the operating points for the spray flow rate, and turbine rotor angular velocity, \( Q_{w0} \text{ m}^3/\text{s} \) and \( n_0 \text{ (rpm)} \) respectively, as a function of current weather, yielding the corresponding net optimal power, \( N_{\text{net0}} \text{ (W)} \) and pumping power \( N_{p0} \).

The task of the feedback controller is to keep \( \Delta n \) zero, where \( \Delta n = n - n_0 \), using \( \Delta Q_w = Q_w - Q_{w0} \), and \( \Delta V_{\text{th}} = V_{\text{th}} - V_{\text{th0}} \), thereby, in view of (15), keeping the net power \( N_{\text{net}} = N_{\text{net0}} \), in response to micro-meteorological wind-induced pressure disturbances, whose spectrum is given in Reference [7]. Such a scheme is feasible if the water storage tanks are sufficiently voluminous to accept temporary deviations from nominal levels until the operating points are adjusted anew with the use of the operating table. In fact, the storage tank water levels may be used as indicators that new operating points are needed.

In this section the simplified linearized model around an operating point is presented. For the detailed development, see Reference [4]. Section 5 contains the design of the feedback controller. By linearizing the turbine equations (12)–(14) around the state operating point \( \begin{bmatrix} n_0 & v_{\text{tur0}} \end{bmatrix}^T \), one gets a second-order transfer function description with constant coefficients. Let the output be \( y(t) = \Delta n(t) = (n(t) - n_0) \), and the input vector \( u(t) = \begin{bmatrix} \Delta V_{\text{th}}(t) \Delta H_{\text{pot0}}(t) \end{bmatrix}^T = \begin{bmatrix} (V_{\text{th}}(t) - V_{\text{th0}}) \end{bmatrix}^T \), hence, in view of (15), keeping the net power \( N_{\text{net}} = N_{\text{net0}} \), with \( Y(s) \) and \( W(s) = \begin{bmatrix} W_1(s) \ W_2(s) \end{bmatrix}^T \) the respective Laplace transforms.

\[
Y(s) = [P_1\mid P_2] W(s)
\]

\[
= \left( s \begin{bmatrix} \frac{60}{2\pi} \mu_j n_0^2 & \frac{60}{2\pi} \mu_j n_0 \end{bmatrix} \right) W_1(s) + \left( \frac{3\mu_j \rho C^2 v_{\text{tur0}}^2}{8\mu_j n_0^2} \frac{\partial v_{\text{pot}}}{\partial H_{\text{pot0}}} \right) e^{-\tau_{\text{pot}}} W_2(s)
\]

\[
\quad + \left( \frac{3\mu_j \rho C^2 v_{\text{tur0}}^2}{8\mu_j n_0^2} \right) \left( \frac{\partial v_{\text{pot}}}{\partial n_0} \right) W_2(s)
\]

where the delay \( e^{-\tau_{\text{pot}}} \) was inserted to reflect the dynamics of the turbine to load power changes.

While \( P_1(s) \) connects \( y \) with a physically manipulable control input, \( \Delta V_{\text{th}} \), the input into \( P_2(s) \) is the intermediate variable \( H_{\text{pot}} \). It is thus necessary to find the transfer function or frequency function from \( \Delta Q_w \) to \( H_{\text{pot}} \), as well as the transfer functions from the assumed pressure disturbances at the top and the bottom, \( \Delta P_{\text{top}} = P_{\text{out}(\text{top}, t)} - P_{\text{out}(\text{top}, t)} \), and \( \Delta P_{\text{bot}} = P_{\text{out}(\text{bottom}, t)} - P_{\text{out}(\text{bottom}, t)} \), respectively, to \( H_{\text{pot}} \). In fact, \( H_{\text{pot}} \) was chosen as a control input in (17) rather than e.g. \( V_{\text{tur}} \) or \( v_{\text{bot}} \) because this enables the decoupling
between the tower and the turbine. Approximate expressions for these frequency functions were
developed in Reference [4], and summarized here.

Assuming that the tower is filled with an ideal gas that undergoes an isentropic process, then
the pressure changes $\Delta P(x, t)$ (Pa) around a stationary point can be modelled with the wave
equation,

$$\frac{\partial^2 \Delta P(x, t)}{\partial t^2} = c^2 \frac{\partial^2 \Delta P(x, t)}{\partial x^2}$$  \hspace{1cm} (18)

where $c$ (m/s) is the velocity of sound. Assume further that the boundary condition at
the top (i.e. equal to the pressure disturbance at the top) is sinusoidal, with frequency
$\omega$ (rad/s),

$$\Delta P_{\text{top}}(t) = \Delta P(0, t) = A_{mp} \sin(\omega t)$$  \hspace{1cm} (19)

The turbine imposes the boundary condition

$$\frac{\partial \Delta P(H_c, t)}{\partial x} = 0$$  \hspace{1cm} (20)

Then (18)–(20) yield that

$$\Delta P(x, t) = A_{mp} \sin(\omega t) \left( \cos \left( \frac{\omega}{c} x \right) + t g \left( \frac{\omega}{c} H_c \right) \sin \left( \frac{\omega}{c} x \right) \right)$$  \hspace{1cm} (21)

From (21) it is easy to find that the resonance frequency is $\omega_{\text{res}} = (\pi c)/(2 H_c) \approx 0.445$ rad/s for
$H_c = 1200$ m. According to Reference [7], the subset of the micrometeorological range in which
the pressure disturbance has a significant contribution is about $0.02$ Hz $= 0.126$ rad/s, and hence
one may assume that $\omega < \omega_{\text{res}}$. Then, one has from (21) that

$$\Delta P(H_c, t) = A_{mp} \sin(\omega t) \frac{1}{\cos(\omega H_c/c)} \Delta P(0, t)$$  \hspace{1cm} (22)

By the use of Bernouilli's equation [10] on which also Equation (13b) is based, one gets

$$\Delta H_{\text{pot}}(t) = \frac{1}{g \rho_a(H_c, t) \cos(\omega H_c/c)} \Delta P(0, t)$$  \hspace{1cm} (23)

Equation (23) can be interpreted that a sinusoidal pressure disturbance of sufficiently low
frequency is amplified with a frequency dependent gain. If the time dependent variable $\rho_a(H_c, t)$
is replaced by an uncertain constant $\hat{\rho}_a$ that covers the range of $\rho_a(H_c, t)$, then one gets the
frequency function

$$\Delta H_{\text{pot}}(j\omega) = \frac{1}{g \hat{\rho}_a \cos(\omega H_c/c)} \Delta P_{\text{top}}(j\omega)$$  \hspace{1cm} (24)

Note that for $\omega = \omega_{\text{res}}$ and its harmonics (24) has infinite gain. In the ‘truth’ model, the gain is
but very large. With $L_D$ (m) the length of the diffuser, one finds in a similar way:

$$\Delta H_{\text{pot}}(j\omega) = \frac{1}{g \hat{\rho}_a \cos(\omega L_D/c)} \Delta P_{\text{bot}}(j\omega)$$  \hspace{1cm} (25)

For the details of the development of the approximate transfer function from $\Delta Q_w$ to $\Delta H_{\text{pot}}$, see
Reference [4]. It is based inter alia on an averaging procedure of the equations in Section 2. With
reference to (4), (6), (9), the use of the approximation $\delta \omega_a \approx -C_{pa} \delta T/L$, where $\delta \omega_a$ (kg water/
kg air) is the change in humidity of the air due to the temperature change $\delta T$ (K), and $C_{pa}$ ($J/kg/K$)
is the specific heat of air at constant pressure, one may obtain the following approximation for
the static dependence of the temperature change $\Delta T$ (K) at the bottom of the tower as a result of a change of the spray flow $\Delta Q_w$ (m$^3$/s),

$$\Delta T = -\frac{d_0^3 A_4 \bar{U}^2}{6Q_{w0}^2 \sigma D_d \delta_d} \left( 1 - \frac{1}{\gamma} \right) g \frac{C_{pa}}{\gamma} \Delta Q_w$$

(26)

where $\bar{U}$ (m/s) is the average air velocity, $\delta_d$ (m) is the average drip diameter, and $\gamma$ is the difference between the temperature gradient for humid and dry air, as in Figure 2. However evaporation is a dynamic process, see (6a), that may be approximated by a linear first-order model and the droplets have a finite velocity, see (6d), which causes a delayed response. One may therefore turn (26) into a dynamic equation in the Laplace domain,

$$\Delta T(x, s) = -\frac{\bar{U}}{Q_{w0}} \frac{1}{\gamma} \frac{g}{C_{pa}} \frac{1}{s + \frac{1}{\tau_2}} e^{-\frac{x}{U_d}} \Delta Q_w(s)$$

(27)

$\Delta T(x, s)$ is the Laplace transform of the temperature change $x$ (m) down the tower, $\tau_2 = \left( \frac{d_0^3 A_4 \bar{U}}{6Q_{w0}^2 \sigma D_d \delta_d} \right)$ (s) is the time constant of the evaporation of the drops, cf. (6b), and $x/U_d$ (s) is the time delay of the droplets. With the help of Bernouilli’s equation, (1) may be turned into an alternative formula for $\Delta H_{pot}$,

$$\Delta H_{pot} = \int_0^{H_t} \frac{-\Delta T(x)}{\bar{T}} \, dx$$

(28)

where $\bar{T} = \bar{T}_{in}$ (K) is the average temperature inside the tower, we get from (27), (28) after integration

$$\Delta H_{pot}(s) = \frac{1}{\bar{T}} \frac{\bar{U}}{Q_{w0}} \frac{1}{\gamma} \frac{g}{C_{pa}} \frac{1}{\tau_2 + s} \left( 1 - e^{-\frac{(H_t/\bar{U}_d)s}{\tau_2}} \right) e^{-\frac{x}{U_d} \Delta Q_w(s)}$$

(29)

where also $\Delta Q_w(s)$ was replaced by $e^{-\frac{x}{U_d} \Delta Q_w(s)}$ in order to represent in a simple way the dynamics of the spraying equipment, whereby $\Delta Q_w(t)$ should from now on be interpreted as the command to the spraying equipment. The model (17), (24), (25), (29) is now complete (see Figure 4).

It can be shown that the plant parameter values for the studied aero-electric power station are such that one of the poles of $P_1(s)$ and $P_2(s)$ is unstable. Let the unstable pole be $-p_1$, and the stable pole $-p_2$. Let $-z_1$ be the zero of $P_1(s)$, $k_1$ the high frequency gain of $P_1(s)$, $k_2$ the high frequency gain of $P_2(s)$, $k_p = k_2/(g\bar{\rho}_0)$, and

$$k_0 = \frac{1}{\bar{T}} \frac{\bar{U}}{Q_{w0}} \frac{1}{\gamma} \frac{g}{C_{pa}} \frac{1}{\delta_d}$$

(30)

With this simplified notation, the block diagram of the plant model (17), (24), (25), (29), (30) is drawn in Figure 5, and its frequency function becomes, with $s = j\omega$,

$$\Delta n(s) = [P_1(s) \quad P_2(s)] \left[ \begin{bmatrix} \Delta N_{in}(s) \\ \Delta Q_w(s) \end{bmatrix} \right] + [P_3(s) \quad P_4(s)] \left[ \begin{bmatrix} \Delta P_{top}(s) \\ \Delta P_{bot}(s) \end{bmatrix} \right]$$

(31)
The set of all operating conditions is characterized by the following set of uncertain parameter ranges (nominal values are underlined):

\[ p_1 \in [2.8, 4.8] \times 10^{-11}, k_2 \in [0.038, 0.29], k_Q \in [1.29, 1.5] \times 10^{-4}, \tau_a \in [0.09, 0.11], \tau_2 \in [3.7, 40], \tau_6 \in [1.8, 2.2], H_c = 1200, \bar{U}_d = [9.5, 19], k_p = k_2/12, L_D = 160, \text{ and } c = 340. \]
4. LOAD SHARING QFT DESIGN FOR REGULATION

The control systems should be designed such that for all operating points the following specifications be satisfied: (i) the closed loop system must be asymptotically stable; (ii) include an integrator such that \( \Delta n(t) \rightarrow 0 \) for step disturbances; (iii) the gain of the output sensitivity function must be less than 2 for all frequencies; (iv) \( |\Delta n(j\omega)/\Delta P_{\text{top}}(j\omega)| < 0.001 \) and \( |\Delta n \times (j\omega)/\Delta P_{\text{bot}}(j\omega)| < 0.001 \) for \( \omega < 1 \) rad/s, excluding a small frequency range around the resonance 0.445 rad/s; (v) minimum use should be made of the control variable \( \Delta N_{\text{in}}(t) \). Specification (iv) implies that all disturbances in the micrometeorological range are damped, as well as disturbances near the first resonance frequency. Specification (v) is meant to ensure that \( \Delta N_{\text{in}}(t) \) will be operated within the capacity of minimal storage tanks.

It is impossible to stabilize the closed loop system with \( \Delta Q_w \) only, because of the combination of an unstable pole, significant delay, and uncertainty. On the other hand it is easy to understand with root locus arguments that it is possible to stabilize \( kP_1(s)/(1 + kP_1(s)) \) with a sufficiently large \( k \) provided \( \tau_a \) is sufficiently small, i.e. globally stabilize the closed loop system with a \( P \)-regulator in the \( \Delta N_{\text{in}}(t) \) loop. In our system \( k = 0.3 \times 10^9 \) was sufficient. The initially closed loop is displayed in Figure 5. The compensated open loop, \( kP_1(s) \), is displayed in a Nichols chart in Figure 6, together with the uncertainty templates for the global set of operating conditions. It follows from Figure 6 that the sensitivity specification (iii) is satisfied. The still
insufficient disturbance rejection is demonstrated in the Bode diagram in Figure 7 of

\[ |\Delta n(j\omega)/\Delta P_{\text{top}}(j\omega)| = |P_3(j\omega)/(1 + kP_1(j\omega))| \]

for the closed loop configuration of Figure 5.

No attempt was made to make the \( \Delta N_{\text{lh}}(t) \) loop satisfy specifications beyond stability and sensitivity, since, specification (v) prescribes that as much as possible of the feedback burden should fall on the \( \Delta Q_w \) loop. It is however important that the system may be globally stabilized by a simple controller in the \( \Delta N_{\text{lh}}(t) \) loop, even if other control loops fail.

According to Reference [8] the ‘slowest’ loop should be closed first, which in the present case is the loop whose control input is the water discharge \( \Delta Q_w \). Closing this loop first will also satisfy specification v. The open conditional water discharge supply plant is given by the transfer function from \( \Delta Q_w \) to \( \Delta n \) in Figure 5, yielding

\[ L_{s2}(s) = \frac{P_2(s)}{1 + kP_1(s)} \] (32)

It was not possible to raise the bandwidth. However, an integrator was included in the controller \( G_d(s) \),

\[ G_d(s) = \frac{0.01(1 + 100s)(1 + 2 \times 0.4s + s^220^2)}{s(1 + 15s)(1 + 20s)^2} \] (33)

thus satisfying specification (ii). \( G_d(s)L_{s2}(s) \) is displayed in Figure 8. See Figure 9 for a block diagram of the system. Figure 10 displays the Bode diagram of \( |\Delta n(j\omega)/\Delta P_{\text{top}}(j\omega)| = |P_3 \times (j\omega)/(1 + kP_1(j\omega) + G_d(j\omega)P_2(j\omega))| \) for the configuration in Figure 9. Clearly specification (iv) is not satisfied. Hence an additional \( \Delta N_{\text{lh}} \) loop stronger than the previously designed P-regulator has to be designed, also to meet specification (iii). The \( \Delta N_{\text{lh}} \) loop could also be made such that specification (iv) be satisfied in a ‘robust’ way, and to increase the bandwidth in order to get a rapid disturbance response.

The open conditional supply loop for the design of the final \( \Delta N_{\text{lh}} \) controller is given as the transfer function between \( \Delta N_{\text{lh}}' \) and \( \Delta n \) in Figure 9,

\[ L_{s3}(s) = \frac{P_1(s)}{1 + kP_1(s) + G_d(s)P_2(s)} \] (34)
Figure 8. The nominal compensated open conditional water discharge supply loop $G_d(s)L_2(s)$ from (32), (33), in a Nichols diagram (dB vs deg), together with Horowitz bounds emanating from the closed loop output sensitivity specification, $|S| < 6$ dB.

Figure 9. Block diagram of the system after the closure of the conditional water discharge supply loop with the controller $G_d(s)$ in (33). $\Delta N_{lh}$ is the control input for the subsequent final control loop.
In order to attenuate $|\Delta n(j\omega)/\Delta P_{\text{top}}(j\omega)| = |P_3(j\omega)/(1 + kP_1(j\omega) + G_d(j\omega)P_2(j\omega))|$ further in the frequency range $[0.0002, 0.03]$ rad/s, cf. Figure 10, the following disturbance rejection specification was introduced:

$$
\frac{\Delta n(s)}{\Delta P_{\text{top}}(s)}_{\text{spec}} \leq \frac{(s + 2 \times 10^{-3})(s + 5 \times 10^{-3})}{(s + 2 \times 10^{-5})(s + 5)}
$$

(35)

The resulting compensator is

$$
G_f(s) = \frac{10^8\left(\frac{s}{10^{-8}} + 1\right)\left(\frac{s}{0.005} + 1\right)\left(\frac{s}{0.2} + 1\right)\left(1 + \frac{2 \times 0.7s}{10} + \frac{s^2}{10^2}\right)}{\left(\frac{s}{10^{-5}} + 1\right)\left(\frac{s}{10} + 1\right)^2\left(\frac{s}{100} + 1\right)\left(1 + \frac{2 \times 0.6s}{0.03} + \frac{s^2}{0.03^2}\right)}
$$

(36)

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Figure 10. Bode gain diagram of $|\Delta n(j\omega)/\Delta P_{\text{top}}(j\omega)| = |P_3(j\omega)/(1 + kP_1(j\omega) + G_d(j\omega)P_2(j\omega))|$ which is the disturbance transfer function for the closed loop configuration of Figure 9. Due to finite numerical resolution, the resonance peak at 0.445 rad/s does not reach infinity. The plant cases cover the uncertainty set.

Figure 11. Block diagram of the final control system. $G_f(s)$ is given in (37), and $G_d(s)$ in (34).
It can be shown that specification (iii) is satisfied. The block diagram of the final system is found in Figure 11. Figure 12 displays the Bode diagram of  
\[ |\Delta n(j\omega)/\Delta P_{top}(j\omega)| = |P_3(j\omega)/(1 + (k + G_f(j\omega))P_1(j\omega) + G_d(j\omega)P_2(j\omega))| \]  for the configuration in Figure 11.

5. SIMULATIONS

Since internal damping and friction are taken into account, the ‘truth’ model of the Aero-Electric Power Station in Section 2 will not exhibit the infinite resonances hinted at in Figures 7, 10 and 13. The control system was checked by the response to a pressure step disturbance

\[ \Delta P_{top} = \begin{cases} 0, & t < 0 \\ 10, & t \geq 0 \end{cases} \]  (37)

in a simulation where the final controller defined in Figure 11, and the plant was defined by the ‘truth’ model in Section 2. The result is seen in Figure 12, which shows that while it is possible to get satisfactory performance with the stabilizing controller in Figure 5, only the advantage of the full controller of Figure 11 is energetic: less water discharge and more pumping power for the low altitude reservoir. Additional simulations are reported in Reference [4].

6. CONCLUSIONS

The general conclusion is that it is possible to design a regulator for the Aero-Electric Power Station that satisfies stringent specifications, using QFT and load sharing, for the full set of operating points. Indeed one might even perceive a certain over-design in this paper.

Another interesting issue is the presence of the resonances at harmonic frequencies, see Figure 7. Damping these is a problem similar to echo cancellation in telecommunication systems or active vibration damping. In this paper no special measures were taken, and the attenuation was aided by the natural damping in the tower. Problems that remain to be solved include, inter alia, design of a non-linear control algorithm, the use of feed-forward from disturbances, and the development of a multi-turbine three dimensional ‘truth’ model and the solution of the subsequent MIMO design problem.
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