Optimal monitoring and management of a water storage

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Abstract The aim is to fill a water storage with potable water of a given quality, for subsequent treatment and distribution to a water conveying system. During a given period, a set of several pumping stations is working to deliver water from different sources at different locations. A multi-stage control process is considered whereby the total pumping time is divided into short sampling intervals. The intensity of pumping as a function of time is the control variable. It is assumed that there exists a reliable forecast of every pollutant as a function of time and water source. The amount of the pollutants are constrained in the final mass of water in the storage. The mass of water at the end of the operation period should be maximized. A linear programming (LP) model of the problem is described, and an algorithm of the reduction of its dimensionality is presented. An illustrative example is shown.

Keywords Pollution · Toxicity · Pathogens · Water management · Monitoring · Optimal control

Introduction

Monitoring and management of pollutants constitute a severe problem for the quality of potable water in freshwater reservoirs, and local natural water sources like lakes. Pollutants are associated with sewage effluents, industrial effluents, waste products from agriculture, and animal wastes from intensive farming. E.g. the main problem associated with certain cyanobacteria is that they can produce toxins that are poisonous to cattle, wildfowl, fish and people. Many species of cyanobacteria have been observed to produce these toxins. The toxins can be divided into three groups: peptide hepatotoxins, neurotoxins and lipopolysaccharides. Among other pollutants levels of cadmium, copper, mercury, zinc, hexachlorobenzene, polycyclic aromatic hydrocarbons and chlorobiphenyls are of especial concern. The sharp decline in the concentration of some heavy metals in water is necessary to slow down close to the quality standards. Methods which are in general expensive, are applied to detect, monitor, and quantify these pollutants and toxins in natural waters, taking into consideration their temporal and spatial variability.
Strictly limited levels of pollutants are required to maintain acceptable water quality, as prescribed by the Pollution Control Agency (PCA) and the Agency for Toxic Substances and Diseases Registry (ATSDR). Acceptable water quality is ensured by checking water quality indicator levels and e.g. comparing them to Maximal Allowable Concentrations (MAC) or Minimal Risk Levels (MRL) at a finite number of locations.

Temporal variations in the magnitude of peak events associated with local concentrations of different pollutants and contaminants and their compounds determine the relative risk to raw drinking water quality, since it is the variations in loads during acute events that are frequently associated with barrier failures in drinking water systems. Thus, predicting the magnitude of event related increases of these loads are crucial to determining the appropriate level of barrier controls necessary to maintain safe drinking water supplies. This is a very important issue for water utility managers and Decision Support Systems (DSS).

Different aspects of water quality monitoring and management and related issues are discussed in many research papers and books, e.g. (World Health Organization 1993; Gardner et al. 2000; Luiten and Groot 1992; Zwart and Folkerts 1990; Hendriks and Pieters 1993; OECD 1993; Silvert 1989; Vrijhof 1984; Van der Gaag 1991; Richardson 1995; Robinson 1989; Adriaanse et al. 1995; Booty et al. 1993; Guariso and Werthner 1989; Sahooa et al. 2005; Ouyang 2005). We are considering here a special optimal control problem which similarly to Gutman (1986), Gutman et al. (1993), Propoi (1963) is reduced to a Linear Programming (LP) problem. A special methodology of Ioslovich and Makarenkov (1975), Ioslovich (1999, 2001) is then used in order to reduce dimensionality and a priori determine control variables that are optimally assigned to zero or to upper bounds. This means that the locations and time periods of the intensive and expensive monitoring could be decreased in an optimal way, while the goal of the optimal management of the water storage could be achieved. Also a list of the pollutants from the initial set that really must be monitored is identified more accurately. The detailed statement of the problem is presented in the next section.

**Problem statement**

During $I$ periods the water from $J$ locations must be pumped to the water storage with full volume $P$ for additional treatment and distribution. The variable

$$0 \leq x_{ij} \leq x_{ij}^u,$$

(1)

corresponds to the amount of the water pumped during sampling period $i$ from the source at location $j$. It means that at each sampling interval all the variables are enumerated correspondingly to existing locations of pumping stations. The correspondent upper bound is denoted as $x_{ij}^u$, $[l/h]$. For each index $n$ of the $N$ monitored pollutants and contaminants that we here for simplicity collectively call pollutants, there is a reliable forecast of concentration at each sampling period and each location, $a_{ij}^n$, $[mg/l]$. It is supposed that the source pollutant concentrations at each sampling period do not depend on amount of previously pumped water. The resulting contents of pollutants in the storage, $v_n$ $[mg]$ must be less than a given limiting bound $l^n$, $[mg]$. According to the technological process of the treatment the remaining volume of the storage is filled with high quality water, so the final concentrations of the pollutants will be less than $l^n/P$. Let us assume that all sampling periods are of the same length $c$, $[h]$. As a result the following set of constraints must hold

$$v_n = c \sum_{ij} a_{ij}^n x_{ij} \leq l^n.$$

(2)

The amount of the pumped water into the storage, satisfying the quality constraints (2) must be maximized, thus

$$F = c \sum_{ij} x_{ij} \rightarrow \max$$

(3)

In this problem the lower index $ij$ is associated with the column, and the upper index $n$ is associated with the row of the matrix of constraints. In a real case there may be as much as hundred sampling periods, about ten locations and more
then thousand monitored pollutants. Therefore the dimension of the formulated LP problem may be about \(1,000 \times 1,000\) with all elements of the matrix of constraints being non-zero. Generally speaking, continuous monitoring is highly recommended though expensive. It is usually also noticed that at some periods and locations, the concentrations of pollutants are very high, and the use of this water is definitely prohibited, while at some other periods and locations, the concentrations are very low and thus should be used for pumping. However, these judgements must be supported by appropriate numerical analysis, which we intend to present.

### Presolving analysis

Despite extended computing capabilities there is still a large difference between solving an LP problem of intermediate dimension and of large dimension. Different methods for solving large-scale LP problems are considered in e.g. Dantzig (1963), Gill et al. (1991) etc. Large LP problems almost always contain a significant number of redundant constraints and variables. This mean that some constraints will never be violated and some variables will definitely be on a zero or maximal bound. Therefore it is much more effective to devote some effort to presolving analysis and considerably reduce the size of the problem. In this way sometimes previously untractable problems can be solved. However, the main reason is that computational resources can be saved significantly and an important information on redundancy of rows and columns, means variables and constraints, can be revealed in advance.

Usually, presolvers are used to reduce the size of an LP problem. Various presolvers are described in Karwan et al. (1983), Brearly et al. (1975), Gould and Toint (2004). Nowadays presolvers are an integral part of many widely used LP-solvers. These presolvers play a significant role in the ability to handle very large size problems. One must keep in mind that, when pushing the button in order to solve a large scale LP, usually, by default, the first step will be a presolving procedure. Presolving analysis of non-negative LP problems with box-constrained uncertainty in the coefficients is treated in Ioslovich (2001). Here for simplicity we shall skip the aspects related to uncertain input information.

Let us consider the general non-negative box-constrained LP problem in the form

\[
\varphi = f'x \rightarrow \max \\
Ax \leq l, \ 0 \leq x \leq xu,
\]

\[
0 \leq l, \ 0 \leq A, \ 0 \leq f, \ 0 \leq xu
\]

\[
A \in \mathbb{R}^{m \times n}, \ l \in \mathbb{R}^{m}, \ x, xu, f \in \mathbb{R}^{n}
\] (4)

All coefficients are assumed to be non-negative. We shall denote the rows of matrix \(A\) as \(a_i'\) and the columns as \(s_j\). The dual problem has the form

\[
\varphi = ly + xu' \rightarrow \min \\
f \leq A'y + u,
\]

\[
0 \leq y, \ 0 \leq u,
\]

\[
y \in \mathbb{R}^{m}, \ u \in \mathbb{R}^{n}.
\]

(5)

Here \(y\) is the vector of dual variables, related to the row constraints, and \(u\) is the vector of dual variables related to the upper bounds of the primal variables. The presolving method in Ioslovich and Makarenkov (1975), Ioslovich (2001) is based on a set of tests dealing with a single row or column. The number of calculations in the single test for one row constraint is of the same order as the problem to find a median of a set of \(n\) real numbers, (Cormen 1990). Let us consider the sets of LP problems in the primal form (4) and in the dual form (5). One can notice that the problem (1–3) is a particular case of these general problem. For these LP problems we have to find guaranteed evaluations that will permit to detect the redundant variables and row constraints.

### General background

The aim of the method is to extract those constraints, that will always be satisfied because of other constraints, and those variables that can be set in advance to its boundary values as a result of column redundancy. For this purpose a number of auxiliary small tests are performed, each of them
being an LP problem with one row constraint and box-constrained variables. These tests make it possible to evaluate the row constraints and to find and remove some of the redundant ones. In the second stage, a similar procedure is applied to the dual problem. This leads to the reduction of the number of variables (columns). Then the first stage is repeated, and the testing procedure becomes iterative. One also can note that as a result of the current reduction, the problem can be decomposed into a set of smaller problems.

First one calculates

\[ l_u = Ax_u. \]  

If for some component \( i \) of the vectors \( l \) and \( l_u \) the inequality

\[ l_i > l_{ui} \]  

holds, then, obviously, row \( i \) of matrix \( A \) is redundant and has to be removed. This simple test is described in Karwan et al. (1983). It is important, however, to assume that this operation already has been done and inequality (7) is not satisfied. Let us consider the auxiliary problem with one linear constraint

\[ \psi = f'x \rightarrow \text{max} \]
\[ d'x \leq b, \]
\[ 0 \leq x \leq x_u, \]
\[ f \geq 0, \quad d \geq 0, \quad b \geq 0. \]  

The solution of the auxiliary problem (8), which is called the “continuous knapsack problem” (CKP), was described in detail in Dantzig (1963, p. 517). The simple proof and the description of the algorithm is presented below in Appendix A. From the optimality conditions, and by denoting the dual variable for the single row constraint as \( \xi \), it follows that

\[ \forall (j: f_j < d_j \xi), x_j = 0; \]
\[ \forall (j: f_j > d_j \xi), x_j = x_{uj}. \]  

The optimal solution will include the variables \( x_{j_1}, x_{j_2}, \ldots, x_{j_p} \), ordered by the decreasing sequence \( f_{j_p}/d_{j_p} \), such that

\[ \sum_{k=1}^{p} d_{j_k} x_{j_k} = b \]  

All the variables, \( x_{j_k} \) except the last one will be set to the upper limit \( x_{uj} \). The last variable \( x_{j_p} \) which corresponds to \( f_{j_p}/d_{j_p} \), becomes the basic variable and is included into the solution with an intermediate value,

\[ 0 \leq x_{j_p} \leq x_{uj} \]  

The value \( f_{j_p}/d_{j_p} \) will be equal to the dual variable, \( \xi \). If the basic variable is not equal to an intermediate value (degenerate case), then it will be assumed that the dual variable \( \xi \) is equal to \( f_{j_p}/d_{j_p} \), where \( p \) is the last value of the sorted index that corresponds to the variable included in the solution which was set to its upper bound. Let us consider the problem of type (8) replacing the constraint \( d'x \leq b \) with a single constraint of the primal problem (4), namely by the row \( i \). This problem will have the following form

\[ \varphi_i = f'x \rightarrow \text{max} \]
\[ a'_i x \leq l_i \]
\[ 0 \leq x \leq x_u. \]  

The dual variable of this problem which is calculated similarly to \( \xi \) will be denoted as \( y_{ui} \), and the vector of \( m \) components \( y_{ui} \) as \( y_u \in \mathbb{R}^m \). The following theorem was proved in Ioslovich and Makarenkov (1975):

**Theorem 1** For the pair of problems (4–5) and the set of problems (12) the following inequalities hold

\[ y_i^* \leq y_{ui}, \quad \forall (i = 1, \ldots, n), \]  

where \( y_i^* \) is i-th component of the optimal solution of the dual problem (5).

Using the upper limits of the dual variables in Eq. 13 one can add box constraints to the dual problem (5). Theorem 1 permits to use the link between tests of the primal and the dual problems. The problem (12) is aggregated, meaning that all row constraints of the problem (4) are summed with non-negative coefficients. All the coefficients of aggregation are zero except the coefficient for the constraint \( i \) which is equal to 1. The aggregated problem has an equal or greater feasible set than the feasible set of the primal problem.
Therefore the optimal value of the objective for the aggregated problem can be used as the upper bound for the objective of the original problem (4).

The objective value of an aggregated problem is the upper bound for the objective of Eq. 4, (Rogers et al. 1991), e.g.

\[ \varphi \leq \varphi_i \]  

(14)

Denoting

\[ \varphi_l = \min_i \varphi_i, \]  

(15)

the following inequality holds

\[ f'x \leq \varphi_l. \]  

(16)

The inequality (16) can be considered as an additional constraint of the problem (4) which may be useful for detecting row constraint redundancy.

**Evaluation of the rows**

Now we shall consider a sufficient condition for row constraint redundancy. The test has following form

\[ \alpha_i^f \leq l_i, \]  

(17)

where the value \( \alpha_i^f \) is the upper bound for the left-hand side of the row constraint \( i \). This value has to be obtained as the solution of an auxiliary LP problem. Finding upper bound for the right-hand side of the constraint, one obtains

\[ \alpha_i^f = a_i'x \rightarrow \max \]

\[ f'x \leq \varphi_l \]

\[ 0 \leq x \leq x_u. \]  

(18)

If the test (17) holds, the row \( i \) is redundant.

**Dual tests**

For the optimal values of the dual variables the following inequality holds

\[ l' y + x'_u u \leq \varphi_l. \]  

(19)

The inequality (19) is the corollary of the equality of the optimal values of the criterion for the primal and dual problems, see Duality Theorem, (Dantzig 1963). Multiplying the inequality in Eq. 5 with the vector \( x_u \) and summing, one obtains

\[ y' l_u + u'x_u \geq f'x_u. \]  

(20)

From Eqs. 19 and 20 it follows that

\[ y'(l_u - l) \geq f'x_u - \varphi_l. \]  

(21)

From Eq. 19 it also follows that

\[ l' y \leq \varphi_l. \]  

(22)

Thus one has obtained two inequalities for the dual variables \( y \), without the dual variables \( u \). The vector of the dual variables that is found from the solutions of the set of problems (12) is denoted as \( y_u \) and used as an upper bound for the dual box-constrained problem.

**Detecting variables at zero bound**

We can solve the problem

\[ \eta_{j\beta} = s_j' y \rightarrow \min \]

\[ y'(l_u - l) \geq f'x_u - \varphi_l \]

\[ 0 \leq y \leq y_u \]  

(23)

Now we obtain the test

\[ \eta_{j\beta} \geq f_j. \]  

(24)

This is a sufficient condition to keep variable \( j \) at zero level, and the column \( j \) can be removed.

**Detecting variables at upper bound**

An additional test can be obtained by solving the problem

\[ \eta_{uj} = s_j' y \rightarrow \max \]

\[ l' y \leq \varphi_l \]

\[ 0 \leq y \leq y_u. \]  

(25)

This test has the form

\[ \eta_{uj} < f_j. \]  

(26)

If the inequality (26) is satisfied then the variable \( j \) must be set to its upper bound. It means that the value of \( x_j \) must be as large as \( x_{uj} \). Thus the vector \( s_j x_{uj} \) must be subtracted from \( l \). In this way these columns are removed and the input data are significantly reduced.
Numerical example

We consider the following illustrative artificial example. There is a water storage system connected with a lake from where five pumping stations pump water during one given day. Hourly pumping periods are considered, thus there are 24 periods and 120 variables. Ten pollutants are monitored. The size of the LP matrix will hence be $10 \times 120$. The full storage capacity in our example is $6,500 \ [l]$. The values of the right hand sides (RHS) of the constraints for the chosen monitored pollutants (4) [mg] are shown below and compared with corresponding MRL (Table of ATSDR):

Acenophanthene MRL = 0.6, RHS = 300
Acetone MRL = 2.0, RHS = 1,000
Acrylonitrile MRL = 0.1, RHS = 50
Aluminum MRL = 2.0, RHS = 1,000
Anthracene MRL = 10.0, RHS = 5,000
Arsenic MRL = 0.005, RHS = 2.5
Chlorodibromomethane MRL = 0.1, RHS = 50
Copper MRL = 0.1, RHS = 50
Kerosene MRL = 0.01, RHS = 5
Mercury MRL = 0.0002, RHS = 0.1  (27)

Suppose that the maximal capacity of each of the 5 pumps is $80 \ [l/h]$. Thus the upper box constraint for each variable $x_{ij}$ is

$$x_{ij} = 80.$$  

The MATLAB program has formed the matrix $A$ and the presolving has been proceeded according to Ioslovich (1999). The matrix is formed in such a way that the pollutants related to the Aluminum are dominant. The presolving program IVITEST (Ioslovich 1999) shows that only 10 out of all 120 variables could assume an intermediate value, 35 variables are fixed on the zero level (no pumping) and 65 variables are fixed on the upper level (maximal pumping). The maximal value of the pumped water volume is $5,879.5 \ [l]$. All the constraints, except the 4th (Aluminum), are redundant. This means that in this lake only the Aluminum concentration must be monitored and there is no need to monitor remaining nine pollutants. The optimal values of all variables are shown on Fig. 1. We remind that each five consequent variables correspond to consequent time interval, 1–24, and in each time interval one variable corresponds to one pumping station.

### Conclusion

It is obvious that the opportunity to determine numerically the time and locations of low and high levels of the pollutants is very important for establishing the optimal monitoring policy, and decreasing the number of the necessary expensive tests. It is also important to point out in advance those pollutants which really have an effect on a safe management policy. The methodology described in this paper is directed toward this problem. In addition, the scale of the LP problem is reduced, thus simplifying the decision making process. The optimal management policy is obtained as a result of the solution of the reduced order LP problem. The MATLAB program for demonstration of the illustrative example can be obtained by e-mail request.

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Appendix

The CKP algorithm

Consider the LP problem (8) noting that it has one linear row constraint only. Introduce the auxiliary vector $Y$ with components $Y_j = d_j x_j$. Hence the constraints are

$$\sum_j Y_j \leq b,$$

and

$$0 \leq Y_j \leq Y_{uj},$$

and the objective is

$$\sum_j \left(\frac{f_j}{d_j}\right) Y_j \rightarrow \max.$$ 

Now order $Y$ by the decreasing sequence $f_j/d_j, Y_1, Y_2, ..., Y_p, ...$. Next chose $p$ such that

$$\sum_{k=1}^{p-1} Y_{uk} < b,$$

and

$$\sum_{k=1}^{p} Y_{uk} \geq b.$$

Obviously the optimal solution of Eq. 8 will be

$$\sum_{k=1}^{p} Y_{jk} = b,$$

whereby

$$Y_{jk} = Y_{uk}, \quad k = 1, ..., p - 1$$

and $Y_{jp} \leq Y_{ujp}$.

References


