1 Introduction

Heat transfer from solid walls to flowing fluids is a topic of extreme scientific interest as well as of immense practical importance. Experimental methods aimed at enhancing it may be costly and time-consuming, hence Direct Numerical Simulation (DNS) of fluid mechanics and heat transfer can be very attractive. DNS can be regarded as a numerical experiment, which may replace its laboratory counterpart, but its numerical and physical accuracy must first be confirmed.

DNS studies of turbulent heat transfer in channel flow were carried out by Kim and Moin [1], Lyons and Hanratty [2], Kasagi et al. [3], and Lu and Hetsroni [4]. Current DNS calculations assume that the wall-temperature fluctuations are zero, and thus cannot account for the existence of a thermal pattern on the solid wall. Such a pattern, often referred to as thermal streaks, was observed and studied in experiments carried out by Iritani et al. [5], Hetsroni and Rozenblit [6], and Hetsroni et al. [7]. These studies showed that the fluctuations are not zero under constant wall heat-flux boundary conditions and that the unsteady heat conduction inside the wall, associated with the turbulent flow, should be allowed for.

Heat transfer calculations, assuming both zero and nonzero fluctuations of the wall temperature, were performed by Kasagi et al. [8] and by Sommer et al. [9]. It was demonstrated that the fluctuations are strongly influenced by the thermal properties and thickness of the wall. Two types of wall boundary conditions were investigated: (1) constant wall heat flux axially, and constant temperature peripherally, and (2) constant wall heat flux both axially and peripherally. In practical applications and in experimental studies, however, these ideal conditions are not likely to occur and the calculations should be verified experimentally.

The object of the present study is, accordingly, experimental investigation of the thermal pattern on the heated wall. Two cases of axially constant heat flux at the wall are studied: whereby the thermal boundary condition along the periphery is close to isothermal and to isoflux, respectively.

The first case is realized if the heating element is made of, say, a thick copper plate, the second one—when it is made of, say, very thin electrical conductive material with low thermal conductivity. It should be noted that in neither case was it possible to realize ideal thermal wall boundary conditions. We will refer to the case of constant heat flux axially and quasi-isothermal peripherally as H1, and to that of constant heat flux axially and quasi-isoflux peripherally as H2.

Experimental Facilities

The experiments were performed in two recirculation test rigs: in a flume and in a rectangular channel. The two-dimensional flume or channel flow offers several advantages for studies of near-wall coherent structures, as flow visualization is then relatively easy. It was shown by Donohue et al. [10] that the flow is fully developed when the channel is over 100 channel depths long. The turbulent structure of Newtonian two-dimensional flume or channel flows has been thoroughly studied. The principal disadvantages compared with pipe-flow facilities are that long high aspect-ratio channels are more difficult to build and the Reynolds number range with a given pump is narrower. As a result, experiments in the present study were conducted up to Re = 20,000.

The flume flow system is the one described by Hetsroni and Rozenblit [6] and only the main hydraulic parameters are covered here. It comprised of a stainless steel open flume 4.3 m long, 0.32 m wide, 0.1 m deep, and water at constant inlet temperature was recirculated in it. The flow depth was 0.037 m; fully developed flow was established in the region beyond 2.5 m downstream from the flume inlet, as was confirmed through the water velocity profile. The distribution of the root-mean-square values of the streamwise velocity fluctuations measured at the center line of the flume, i.e., at z = 0 (z is the spanwise direction, x the streamwise direction, and y normal to the bottom). The heated test section was located at a distance of 2.5 m from the inlet.

For the H1 conditions we used a section made of a copper plate 0.30 m long, 0.15 m wide, and 0.02 m thick. The temperature distribution over the heated surface was measured from the outer side by 18 thermocouples, and from the liquid side by a Liquid Crystal Sheet (LCS). In the present study, Teflon coated T-type thermocouples (diameter 0.3 mm) were used. Thermocouples were peneed into small holes drilled in the test section. The local outer-wall temperature was obtained from the temperature from the temperatures measured by thermocouples using equation for a plane wall. Since the temperature drop through the wall was very thin electrical conductive material with low thermal conductivity. It should be noted that in neither case was it possible to realize ideal thermal wall boundary conditions. We will refer to the case of constant heat flux axially and quasi-isothermal peripherally as H1, and to that of constant heat flux axially and quasi-isoflux peripherally as H2.

Effect of Constant Heat Flux Boundary Condition on Wall Temperature Fluctuations

An experimental study of the wall temperature fluctuations under different thermal-wall boundary conditions was carried out. Statistics obtained from the experiments are compared with existing experimental and numerical data. The wall temperature fields are also examined in terms of the coherent thermal structures. In addition the effect of the thermal entrance region on the wall temperature distribution is also studied. For water flow in a flume and in a rectangular channel, the mean spacing of the thermal streaks does not depend on the thermal entrance length and on the type of thermal-wall boundary conditions. The wall temperature fluctuations depend strongly on the type of wall thermal boundary conditions. Overall, the picture that emerges from this investigation confirms the hypothesis that moderate-Prandtl-number heat transfer at a solid wall is governed by the large-scale coherent flow structures. [DOI: 10.1115/1.1345886]

Keywords: Boundary Layer, Channel Flow, Forced Convection, Heat Transfer, Temperature, Turbulence

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small, the thermal conductivity of the wall was assumed independent of the normal direction and was evaluated at the local measured temperature. The operational procedure for the water tests without heating showed that the wall temperatures always agreed with ±0.1 K. Heating was effected by means of cartridge electrical elements. The details of the test section are shown in Fig. 1.

The channel flow system is shown in Fig. 2. The 7.2 m long, 0.2 m wide, and 0.02 m deep rectangular channel comprised, twelve Plexiglas sections 0.6 m each carefully joined to ensure a hydraulic smooth surface throughout. Water from a tank with a heat exchanger was delivered by a pump, passed through a control valve, flow meter and flow straightener to a development section, provided for hydrodynamical and thermal development of the boundary layer. The temperature measurements were carried out in the test section. The heating strips 0.6×0.2 m each were installed at the top inside the channel from the front end of the development section to a distance of 0.5 m beyond the test section. These strips were made of 0.05 mm thick stainless steel and arranged so that the boundary layer could be heated along different distances from the inlet to the test section. The latter was provided with two 0.2×0.16 m windows to which the strips were bonded with contact adhesive and coated on the air side with black mat paint about 0.02 mm thick.

For the H2 condition DC current up to 300 A was applied to the heating strips, and measurements were taken at different lengths of the heated stretch. The test section is shown in Fig. 3.

Measurement Techniques

For measuring the temperature field from the water side under the conditions H1, light from a halogen lamp was directed onto the liquid crystal layer. Depending on the temperature the liquid crystal displays different colors. Before experimental runs the color was calibrated versus the temperature, with the same illumination as for the experimental runs. The color play was recorded by a 3CCD video camera and analyzed by means of a specially devised software package. The 35C1W liquid crystal sheet used in the present study yields the color play at the 35–36 °C level with a minimum detectable temperature difference of 0.1 K.

For the H2 conditions a Thermal Imaging Radiometer was used with a typical horizontal and vertical resolution of 256 pixels per line. Since the heating strip was very thin (0.05 mm), the temperature difference between its surfaces did not exceed 0.1 K, Hetsroni and Rozenblit [6]. A computer program made it possible to store the information and to compute the statistics of the thermal field.

The response times were 0.4 s, 0.05 s, and 0.04 s for the measurements by thermocouples, liquid crystal sheet and infrared radiometer, respectively. The different system (liquid crystal sheet and infrared radiometer) used for the measurements of surface temperature fluctuations had about the same frequency response. The water temperature was measured by a precision mercury thermometer with accuracy 0.1 K, the pressure drop was measured by a Motorola pressure transducer with accuracy ±1.5 percent, the mean flow velocity was measured with accuracy ±1 percent, the electric power was determined with accuracy ±0.5 percent.

Data Reduction and Experimental Conditions

The local heat transfer coefficient $\alpha_x$ is defined as

$$\alpha_x = \frac{q}{t_{wx} - t_{fx}}$$

where $q$ is the heat flux, $t_{wx}$ the local inner wall temperature, $t_{fx}$ the mean fluid temperature at the longitudinal position. The calculated Reynolds number is $Re = UH/v$ and $Re = UD_h/v$ for the flume and the channel, respectively, $U$ being the bulk velocity at a cross section of the flume or channel, $H$ the flow depth in the flume, $D_h$ the channel hydraulic diameter, $v$ the kinematic viscosity. The dimensionless thermal entrance length was calculated as
heat flux.

The total heat losses were about 1 percent for the stainless steel heater and 2–3 percent for the copper heater, depending on the test section.

The heat balance for the variation of the outside fluid and wall temperature also increased linearly in the streamwise direction. The heat transfer in the axial direction was calculated from the wall temperature distribution in the streamwise direction. The heat conduction for the heated test section was heated. At $x^*>10$ it is assumed that the bulk fluid temperature increases linearly along the test section. The test measurements indicated that in this case the wall temperature also increased linearly in the streamwise direction.

Throughout this study the distances are also normalized using the ‘inner variables’, $u^* / u_{\infty}$, where $u^*$ is the shear velocity $u_{\infty} = (\tau_w / \rho)^{0.5}$, $\tau$ the shear stress, $\rho$ density. The method for the evolution of the shear velocity in the flume involved fitting the velocity profile, in the logarithmic portion of the profile, to the ‘universal’ velocity profile. For channel flow shear velocities were calculated from the pressure drop measurements. The root mean square of the wall temperature variance $\theta^* = (\overline{\theta^2})^{0.5}$ is normalized by the friction temperature $\theta^* = q / \rho c_p u_{\infty}^*$, (where $c_p$ is specific heat at constant pressure), so that the dimensionless temperature fluctuation is $(\overline{\theta^2})^{0.5} / \theta^*$, where $\theta$ is the fluctuating wall temperature. The heat transfer interaction between the fluid and a solid may be characterized by the thermal activity ratio $K = [(\nu c_p) / (\nu c_p + k)]^{0.5}$, the dimensionless wall thickness $\delta^* = \delta^* (\alpha_w / \alpha_s)^{0.5}$ Kasagi et al. [8], and the Prandtl numbers $Pr_f$ and $Pr_s$. Here, $k$ is the thermal conductivity, $\delta^* = (\delta u^*) / u^*$, $\delta$ the wall thickness, a thermal diffusivity, the subscripts ‘$f$’ and ‘$w$’ denote properties at the fluid and wall temperature, respectively. The experimental conditions are listed in Table 1.

Test Procedure

Heat Losses. The axial heat conduction for the heated test sections was calculated from the wall temperature distribution in the streamwise direction. The heat transfer in the axial direction was less than 0.5 percent of that in the wall normal direction for the thin stainless steel heater, and less than 1 percent for the thick copper heater. The heat balance for the variation of the outside ambient temperature was also verified by direct measurements. The total heat losses were about 1 percent for the stainless steel heater and 2–3 percent for the copper heater, depending on the heat flux.

Experiments in the Flume. It was shown earlier by Hetsroni et al. [11], that the distribution of the dimensionless streamwise velocity versus the dimensionless wall-normal distance in the flume is in good agreement with the measurements presented by Antonia et al. [12], and the distribution of the turbulence intensity agrees well with the data by Nishino and Kasagi [13]. The experimentally obtained heat-transfer coefficients are in accord with the predictions of Kays [14]. In the present study we determined the temperature distribution on the heated surface from the water side with an LCS and from the solid side with thermocouples. In both cases the standard deviation of the local wall temperature from the space and time averaged surface temperature did not exceed 0.2 K in the heat flux range $q = 10–12$ kW/m$^2$. The thickness and high thermal conductivity of the copper plate used in the present study allows to assume that the wall temperature is nearly constant. For experiments without liquid crystal sheet, at $Re = 5200$ and $q = 10^4$ W/m$^2$ the mean wall temperature was $28.7 \degree C$, the mean water temperature was $20.0 \degree C$. In this case the maximum increase of the wall temperature in the streamwise direction does not exceed 0.3 K and the estimated conduction heat rate in the axial direction was about $1.2$ W, while heat rate in the wall normal direction was $450$ W.

It should be noted, in the present study the ‘isothermal boundary condition (H1)’ means that the wall temperature does not change in the streamwise direction. Expression ‘isoflux boundary condition (H2)’ means that the wall temperature fluctuations are not zero. It will be shown that the copper heating plate provides very small wall temperature fluctuations. It was believed that the H1 thermal wall boundary condition existed.

Experiments in the Channel. A number of verification runs were undertaken prior to data logging. Figure 4 shows the Reynolds number dependence of the wall-shear velocity normalized by the bulk (mass average) velocity. The data obtained in the present study are in good agreement with the results reported by Donohue [10].

Figure 5 shows the dependence of the dimensionless local heat transfer coefficient $\alpha_{fl} / \alpha_s$ on the dimensionless distance, $x$, from the start of heating at $Re = 17000$. Here $\alpha_s$ is the experimental asymptotic heat transfer coefficient. The data of Hartnett [15] for water heat transfer at $Re = 16,900$ are also shown for comparison. The good agreement between these experimental results is established.

The distinct feature of these experiments is that the rms of the wall temperature fluctuations in the H2 case is about an order higher than in the H1 case.

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</table>

Fig. 4 Wall-shear velocities in channel water flow [●-present study; ○-data of Donohue [10]]

Fig. 5 Dependence of dimensionless local heat transfer coefficient on dimensional thermal entry length [●-present study; ○-data by Hartnett [15]]
Results

Macro-Scale Thermal Structure. As pointed out, appearance of the macro-scale thermal structures on the wall in a turbulent flow was observed previously. These structures (so-called thermal streaks) are oriented mainly in the streamwise direction, moving slowly downstream and meandering about in the lateral direction, with some twisting and merging with the neighboring coherent flow structures. No experimental study seems, however, to be available on their behavior with allowance for the unsteady heat conduction in the solid wall and the thermal entrance region.

The temperature distribution on the heated wall can be considered as a trace of the flow structure there. The streak spacings were obtained by infrared or liquid crystal sheet visualization technique. The mean spanwise spacings were determined by examining the two-point correlations of the temperature fluctuations in the spanwise direction as in the case of the low-speed streaks [1].

Thermal Streaks Spacing Under Isothermal (H1) Wall Boundary Condition. The dimensionless spacing of the thermal streaks \( \lambda^+ \) versus the dimensionless thermal entrance length is shown in Fig. 6, where \( \lambda^+ = \lambda u^*/\nu \) (\( \lambda \) being the average streak spacing at a given value of \( \bar{\lambda} \)). One can conclude that the value of \( \lambda^+ \) does not change within the measurement uncertainty of \( \Delta \lambda^+ = \pm 5 \).

Thermal Streaks Spacing Under Isoflux (H2) Wall Boundary Condition. Figures 7 and 8 show the behavior of \( \lambda^+ \) in the thermal entrance and developed regions, respectively. In the present case two-point correlations indicate \( \lambda^+ = 95 \) (Fig. 7) and \( \lambda^+ = 97 \) (Fig. 8). From Figs. 6, 7, and 8 one can conclude that in both H1 and H2 thermal wall boundary conditions \( \lambda^+ \) is about 100. This value is in close agreement with the results presented by Iritani et al. [5] and Hetsroni et al. [7]. The well-known experiments established that the mean streak spacing between near-wall coherent flow structures is also about 100 wall units, a result confirmed by the high-resolution DNS of Kim and Moin [1]. They showed that at Prandtl number \( Pr = 0.71 \), the temperature in the near-wall region is highly correlated with the streamwise velocity. The present experiments were carried out at Prandtl number \( Pr = 5.5–5.9 \) and in this case the spacing is also about 100 wall units. This result is consistent with the experimental observation of Iritani et al. [5], who performed simultaneous visualizations of the velocity and temperature fields in a turbulent boundary layer of water flow, using hydrogen bubbles for the velocity field and a surface-mounted liquid-crystal sheet, sensitive to the wall temperature, for the scalar field. Kim and Moin [1] showed that although the sublayer of the thermal boundary layer is a strong function of the Prandtl number, it appears that thermal streaks in the wall region are almost independent of the molecular Prandtl number.

Wall Temperature Fluctuation

Figures 9 and 10 show the wall temperature variation in the spanwise direction for H1 and H2 case respectively. One can see that in the case of H2 the temperature fluctuations are higher. We will discuss this issue below.
**H1 Wall Temperature Boundary Condition.** The plot of the dimensionless wall temperature fluctuation on the heated wall, \( \theta^+ \), versus the dimensionless thermal entrance length, \( \bar{x} \), is given in Fig. 11. As mentioned above, wall temperatures were determined both by liquid-crystal sheet and by thermocouples, with good agreement between these two methods. Figure 11 shows that the behavior of the wall temperature is independent of the thermal entrance length.

Before closing this section, let us add a remark on the behavior of \( \theta^+ \) versus the thermal entrance length. Under fully developed thermal and hydrodynamic conditions, temperature fluctuations are a consequence of turbulent velocity fluctuations of the flow and are closely correlated with its structure, depending on the Prandtl number and the distance from the wall. However, due to the thermal interaction of the flow with the wall, there also arise temperature fluctuations. Therefore, the problem concerning the temperature fluctuation of the liquid near the wall should be considered in a related manner. To-date, scant attention has been paid to the effect of the thermal entrance length in this context. The results of the present study indicate that the wall temperature fluctuation in the thermal entrance region in the H1 case is similar to isothermal wall. This conclusion agrees quite well with the experimental results of Hishida and Nagano [16], where the test section was heated by saturated steam under atmospheric pressure, thereby ensuring a uniform wall temperature. It was established that in the inner part of the boundary layer the overall characteristics of the temperature fluctuations and the correlations between it and the flow velocity are almost identical throughout the thermal entrance region.

**H2 Wall Temperature Boundary Condition.** The wall temperature fluctuations for the thermal entrance region and for the developed thermal boundary layer are shown in Figs. 12 and 13 respectively. The \( \theta^+ \) versus \( \bar{x} \) pattern in the first figure indicates that wall temperature fluctuation lessens in the streamwise direction, while the second figure shows that in the developed thermal boundary layer the wall temperature fluctuation is uniform, but at a much higher level than in the H1 case. This means that the effect of the thermal wall boundary conditions on the wall temperature fluctuation is very strong. The present experimental results have been compared with the numerical calculations of Kasagi et al. [8], showing reasonable agreement with numerical calculation. Unlike with the H1 case, no experimental study seems to be available on wall temperature fluctuations in the H2 case. In the experiments of Subramanian and Antonia [17] an electrically heated stainless-steel foil served as the wall surface in the turbulent boundary layer. As was shown by Kasagi et al. [8] in air flow (in particular, under the conditions of the above experiments) the wall temperature fluctuation should be negligibly small, so that the wall can be regarded as ideally isothermal. The experiments of Slanciauskas et al. [18] were made in a two-dimensional water channel flow with 0.8 mm thick stainless steel foil attached to a 5 mm thick Plexiglas board and heated directly by an electric current. Here again the heated surface cannot be regarded as an ideal isoflux wall, since the heat transfer took place through a thick board by thermal conduction. At the same time, the H2 case of thermal wall boundary condition is very important in validating new methods of numerical calculation of a passive scalar distribution in a turbulent velocity field. For every location of \( \bar{x} \), the intensity of the velocity fluctuation decreases as the wall is approached, so that in the H2 case the temperature field becomes less correlated with its velocity counterpart in the immediate vicinity of the wall.

In the present study we did not observe any difference between the heat transfer coefficients obtained in the H1 and H2 thermal wall boundary conditions. In other words, the H2 condition tend to increase \( \theta^+ \) only, while the normal turbulent heat flux is essentially independent of the wall temperature fluctuation. This was also verified by the calculations of Sommer et al. [9]. The influence of the isothermal and isoflux temperature boundary condition is important for the analytical approximation and heat transfer calculations. These two types set the limits for the conjugate heat transfer calculations. Effect of the thermal boundary condition is very important in the near-wall region, i.e., at high Prandtl numbers. As was pointed out by Kasagi et al. [8], the thermal activity ratios for air flow is very small for all practical wall materials. On the other hand, thermal activity ratios for water flow are large enough, and in the combination with the varying wall thickness and heat conductivity almost isothermal and isoflux wall temperature conditions can be achieved in experiments and in engineering devices. Value of wall temperature fluctuations is the crucial parameter for the resolution requirements for DNS at high Prandtl numbers.

### Conclusions

- The spacing of the thermal streaks on the wall is independent of the thermal entrance length.
- The wall temperature fluctuations at a constant wall heat flux axially and a constant temperature peripherally (H1) is also independent of the thermal entrance length.
- The wall temperature fluctuations at a constant wall heat flux axially and peripherally (H2) lessens in the streamwise direction in the thermal entrance region.
• The wall temperature fluctuation depends strongly on the type of wall temperature boundary condition: its rms under the H2 wall boundary conditions is about an order of magnitude higher than that under the H1.

• For heat transfer from the solid wall in water flow almost isothermal or isoflux boundary condition can be achieved, depending on wall thickness and its thermal conductivity. At high values of heat flux for the wall temperature oscillations may be, in fact, more damaging, than high absolute wall temperatures. Knowledge of the value of wall temperature fluctuations is very important for designing and operation of engineering devices.

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Nomenclature

\[ a = \text{thermal diffusivity} \]
\[ c_p = \text{specific heat of the fluid at constant pressure} \]
\[ D_h = \text{channel hydraulic diameter} \]
\[ H = \text{flow depth} \]
\[ K = \text{thermal activity ratio} \]
\[ k = \text{thermal conductivity} \]
\[ Pr = \text{Prandtl number} \]
\[ q = \text{heat flux} \]
\[ Re = \text{Reynolds number} \]
\[ T = \text{temperature} \]
\[ t_{fs} = \text{mean fluid temperature at the longitudinal position} \]
\[ y_{wa} = \text{local inner wall temperature} \]
\[ U = \text{bulk velocity} \]
\[ u^* = (\tau_0/\rho)^{0.5} = \text{friction velocity} \]
\[ x = \text{streamwise distance} \]
\[ y = \text{distance from the wall} \]
\[ z = \text{spanwise distance} \]
\[ \bar{x} = x/H, \ x/D_h = \text{dimensionless thermal entrance length} \]
\[ \alpha_x = \text{local heat transfer coefficient} \]
\[ \delta = \text{wall thickness} \]
\[ \delta^+ = \delta u^*/v = \text{dimensionless wall thickness normalized by the boundary layer parameters, } u^* \text{ and } v \]
\[ \delta^{+ \ast} = \delta^+(a_1/\alpha_w)^{0.5} = \text{dimensionless wall thickness normalized by ratio of thermal diffusivities} \]
\[ \lambda = \text{thermal streaks spacing} \]
\[ \lambda^* = \lambda u^*/v = \text{dimensionless thermal streaks spacing} \]
\[ \rho = \text{density} \]
\[ \nu = \text{kinematic viscosity} \]

\[ \theta = \text{wall temperature fluctuations} \]
\[ \theta^* = q/pc, u^* = \text{friction temperature} \]
\[ \theta^* = (\theta^*)^2/\theta^* = \text{dimensional RMS of the wall temperature fluctuations} \]
\[ \tau_w = \text{shear stress} \]

Subscripts

\[ w = \text{wall} \]
\[ f = \text{fluid} \]

References