Shear-induced corrugation of free interfaces in concentrated suspensions

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Abstract
When a concentrated suspension of small inertia-less particles in a viscous fluid is sheared between two parallel-belts a free surface exists with a direction orthogonal to the plane of the simple shear flow. We observed a development of corrugation on this surface. The corrugation appears as an ensemble of disturbances driven by shear-induced diffusion and restrained by surface forces. The roughness of the surface disturbances depends on the particle size, on particle concentration and on the fluid surface tension. The corrugation has many lengthscales above and below particle size and it appears in the form of waves traveling with the local surface fluid speed.

In this communication, we present a report on the phenomenon and its dependence on the physical parameters. A qualitative description is presented and the dimensionless groups governing the extent of the shear-induced corrugation are defined. We introduce the notion of a macroscopic effective surface tension, which depends on the surface particle concentration, with possible influence on the stability of the surface flow. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction
When a suspension of small particles in a viscous fluid is subjected to a shear flow the characteristics of the flow can be significantly different from those of an equivalent homogeneous liquid. The existence of particles in the fluid affects properties such as density, viscosity, thermal conductivity and many others. In many situations it is enough to define effective properties and bulk balances to adequately describe the motion. However, when particles in the suspension are present in a considerable volume fraction, the hydrodynamic interaction between them may significantly alter the flow characteristics and render the definition of average effective properties and their application in macroscopic balances almost useless. For suspensions with micron size particles, for which inertia effects and Brownian diffusion can be neglected,
the interaction between the particles adds a random component to their motion that is additional to the
deterministic translation along streamlines in the slow viscous environment. This random component
results in migration of particles, which was first identified by Leighton and Acrivos [1]. It is known as
shear-induced diffusion, and has attracted much interest ever since.

The only necessary requirements for shear-induced diffusion of particles are two: the existence of a
shear field in the suspension that promotes particles movement relative to others, and the presence of
particles in a sufficient volume fraction that results in hydrodynamic interactions between the particles. If
these interactions are not periodic or if they are irreversible, cross streamline drift of particles can occur.
Thus, Leighton and Acrivos [1] proposed a phenomenological model for the shear-induced particle flux
along gradients of the shear stress intensity and the particles volume concentration. A similar model based
on concentration and on the particles’ collision frequency, itself depending on the shear rate intensity,
was suggested by Phillips et al. [2]. Nott and Brady [3] related the particle flux to gradients in the bulk
pressure in the suspension, which contains a contribution due to the presence of the particles. They also
discussed the possible dependence on effective normal stress differences when they exist. Shauly et al.
[4] suggested a phenomenological particle migration potential as a driving force to the drift that was also
extended to describe migration and segregation in poly-disperse systems.

Much attention was devoted to the effect of the shear-induced discussion phenomenon on various bulk
flow characteristics. Since, in general, flow fields do not possess uniform shear, the motion results in redis-
tribution of particles in the fluid. Consequently, non-uniform particle concentration distributions emerge
and they affect local effective properties such as the effective viscosity. These, in turn, change the distribu-
tion of stress in the system and, as a result, the flow pattern changes as well. Thus, bulk velocity profiles in
the presence of shear-induced migration differ considerably from those akin to the flow of a Newtonian ho-
mogeneous fluid, under similar boundary conditions, and in general the flow of concentrated suspensions
can not be described by that of an equivalent homogeneous fluid having constant effective properties.

The effect of shear-induced particle diffusion on free interfaces received only limited attention so far.
Tirumkuhalu and co-workers [5,6] studied the shearing of concentrated suspensions between two cylin-
ders when the gap was not completely filled, and the rolling of a suspension film on the inner wall of
a rotating partially filled cylinder. In both studies, they observed a migration of particles perpendicular
to the direction of motion and to the plane of shear. The migrating particles accumulated in surface
bands, oriented in the direction of flow, leaving the regions between those bands depleted of particles.
The non-homogeneous distribution practically caused an elevation of the free surface where particles
accumulated and where the effective viscosity was increased. These observations inspired an attempt to
describe the free surface flow as exhibiting an instability [7]. However, the attempted two-dimensional
linear stability analysis yielded results with only remote correspondence to the experimental observations.

In our laboratory, during an attempt to measure particle velocity fluctuations in a uniform stress field
(hoping to relate them to particle self-diffusion in the absence of concentration and shear stress gradient),
we observed a new surface phenomenon. At a certain particle size and particle concentration the surface
became visibly corrugated. As this was not observed in the absence of particles, we concluded that
it is a particles related phenomenon and that it is driven and being sustained by particle shear-induced
diffusion. Hence, in this communication we present the phenomenon of shear-induced surface corrugation.
In Section 2, we present the experimental system. The observed results are depicted in Section 3 in a visual
form as well as by spectra depending on time and space. A diffusion of the observation follows in Section
4 where an analysis of the dimensionless parameters governing the corrugation is presented and ideas
concerning the influence of the appearance of particles at the surface on the effective interfacial tension.
2. Experimental system and procedures

2.1. Apparatus and suspension materials

A parallel-belt device was built to create a flow field which approximated a two-dimensional plane Couette flow. A sketch of the apparatus is shown in Fig. 1. The apparatus consisted of two stainless steel belts, each of them lead around two cylindrical pulleys. The pulleys were placed at the corners of a rectangle of length 20 cm and a width such that the near sides of the belts were located 2 cm apart. The belts were placed in an acrylic box. The inner walls of the box were formed in a way that the distance between the belts and the wall equaled everywhere half the spacing of the belts. This was done to make sure that the shear exerted on the test fluid was nearly uniform throughout the flow region. Slight shear rate gradients were expected only at the small sides of the box where the fluid was conveyed around the pulleys to the far side of the belts. To realize a two-dimensional flow the test fluid was suspended on a layer of a heavy lubrication fluid of low viscosity. In practice, the apparatus was first filled with the lubrication fluid. Then, the test fluid or the suspension was cautiously poured into the test section, on top of the lubrication layer. The interface between the lubrication layer and the test fluid was situated above the lower rim of the steel belts. Therefore, test fluid could not enter the space at the inside of each belt, where the pulleys were located and engaged with the belts. It was an important feature of the apparatus that the lubrication layer effectively sealed off the space at the inside of the belts from the test fluid. In most cases the test fluids consisted of a suspension of oily components and particles, both of which would have interfered unfavorably with the belt drive. The pulleys were smooth and drove the belts by friction only.

![Fig. 1. Sketch of the apparatus. View along section AB and top view.](image-url)
Suspensions were prepared from spherical PMMA-particles and a Newtonian organic liquid obtained from Cargille Laboratories (Cedar Grove, NJ). The liquid, code TR11510, had a density of $\rho = 1.18 \text{ g/cm}^3$, a refractive index $n_D = 1.491$, a viscosity of approximately 200 cSt and a surface tension $\gamma = 46 \text{ dyn/cm}$ at 25 $^\circ$C. Hence, the density and refractive index were matched. For the lubrication layer Fluorinert™ Electronic Liquid FC™-77 (3M Corporation) with $\rho = 1.76 \text{ g/cm}^3$ and a kinematic viscosity $\nu = 0.7 \text{ cSt}$ was used.

Fig. 2. Sketch of the set-ups for (a) the visualization system and (b) the measurement.

Fig. 3. Stripes reflected on the surface of a suspension. Particle concentration 13%, particle diameter 655 μm. The width of the field of view is 1.05 cm. The upper edge of the image shows the surface close to the belt, the lower edge is below the center line between the two belts. The suspension is (a) at rest and (b) sheared with 2 s$^{-1}$. 
Mono-disperse, neutrally buoyant suspensions were prepared from particles of diameters 49±5, 64±11 and 655±55 μm, respectively. The particle concentrations were varied between 13 and 50%.

When the suspensions were sheared, corrugations of the surface appeared. Two different systems were set-up to visualize and to measure some properties of the surface corrugation, respectively. For visualization two different grids were placed above the surface and the reflection of the grid on the surface was filmed with a CCD camera. For the larger particles (655 μm) we used parallel stripes with

Fig. 4. Reflection of a mm-grid on (a) clear fluid (silicone oil, 5000 cSt) and a suspension in Cargille fluid with particle concentration and particle diameter (b) 1%, 655 μm, (c) 22%, 49 μm and (d) 40%, 49 μm. The shear rate is 2 s⁻¹ in all cases.
1 mm spacing between them. For the smaller particles (49, 64 μm) we employed a rectangular mm-grid. A schematic of the set-up is shown in Fig. 2a. Any corrugation of the surface changes the optical path and can be seen as a distortion of the reflected stripes or grid. For finer particles, the distortion blurs the grid. Images of reflected grids are shown in Figs. 3 and 4.

The frequencies and the wavelengths of the corrugation were detected by placing the camera vertically above the surface. The suspension was illuminated with white light from below in a configuration similar to a dark field illumination (see Fig. 2b for a schematic of the set-up). The light beam just passed by the camera lens, if there were no refracting objects in the optical path the field of view for this case would remain dark. The light, however, became more diffuse when passing through the suspension and resulted in a diffusive illumination of the upper surface. Depending on the orientation of the surface the light was refracted and, therefore, the intensity varied according to the surface orientation.

Typical images, similar to those used to acquire data, are shown in Fig. 5. The figure depicts two top views of a 40% suspension sheared at a rate \( G = 2 \text{s}^{-1} \). The two views correspond to the larger (655 μm) and the smaller (49 μm) particle size. The different scales of the surface corrugation are clearly evident. Note that the exact relation between surface orientation and image intensity is unknown. Thus, this technique could not be used to obtain information about the corrugation amplitude. Nevertheless, the image data allows to extract spatial and temporal frequencies.

The imaging system consisted of a monochromatic 10 bit CCD camera (Kodak Megaplus Model ES 1.0) with a resolution of 1008 x 1018 pixels and a maximum frame rate of 30 frames s\(^{-1}\). Either a 105 or a 135 mm lens was used, both were used with extension rings. From a working distance of approx. 50 cm the field of view was approximately 4 cm x 4 cm. The camera was connected via a frame grabber board (Road Runner, Bit flow, Inc.) to a PC with 256 MB RAM. A sequence of images could only be captured to memory, not directly to the hard disc. Since the size of a full image is approx. 1.2 MB, the number of

Fig. 5. View of the surface from top. Particle concentration 40%, shear rate 2 s\(^{-1}\) and particle diameter (a) 655 μm and (b) 49 μm. In (a) the belts can be seen at the sides. The left belt runs upwards, the right belt downwards.
images which could be taken in one run was severely restricted. To ease the memory restriction, slightly different procedures were devised for capturing image sequences, depending on whether frequencies or wavenumbers were to be measured. For spatial measurements the image should have the largest possible extent in the direction of interest. For frequency measurements a long duration of the sequence is desired, while the frame rate must be larger than twice the expected maximum frequency. The frame grabber board was configured in such a way that only part of the image was acquired. In all cases, the images were cropped to the area of interest between the belts. For temporal measurements, a frame in the form of a small stripe of 100 pixels height which extended from one belt to the other was used, see Fig. 6a. After preliminary experiments a frame rate of 15 s$^{-1}$ was found sufficient. 1024 frames were taken, which yield a duration of 68 s for one take. By looking at a video of the sequence, the location of zero velocity was determined visually, with an accuracy of ±2 pixel. Within an area of 5 × 100 pixels a power spectral density (PSD) estimate was computed by a periodogram technique for each pixel. The 500 PSDs obtained were averaged, which resulted in one PSD distribution for a given experimental run. For spatial measurements in the flow direction, a frame of the full height, 1018 pixels, and a width to cover the area between the belts was used (see Fig. 6b). The frame rate was set to 1 s$^{-1}$ and 20 frames were taken. The zero velocity location of the observation area remained the same as determined from the longer sequences.
used for temporal measurements. The spatial PSD perpendicular to the flow direction was computed for five lines and the $5 \times 20$ PSDs from the sequence of 20 frames were averaged. Again, one averaged PSD distribution was obtained for each experiment. Additionally, temporal and spatial auto-correlation functions were computed from the same data as the spectra.

2.2. Flow field, visualization and data acquisition

In Fig. 7, we depict the streamline pattern anticipated in the designed apparatus that was calculated using the finite elements software FIDAP available commercially. The numerical results predicted the flow everywhere to be a simple shear flow with uniform shear, except in small circulation regions near the entrances to (and the exit from) the gap separating the belts. The size of these regions was minimized by

![Fig. 7. The flow field obtained by (a) a numerical simulation and (b) LDV-measurements taken in the middle of the gap. (Δ), (+) and (⊙) correspond to profiles measured at 1.7, 3.84 and 5.7 mm below the surface.](image)
a careful curved design of the boundary as is depicted in the figure. Single clear fluid runs were made to determine the two-dimensional flow field in the apparatus. In these runs, we used the suspending Cargille fluid and a silicone oil that had a higher viscosity, similar to that of the concentrated suspension. In Fig. 7, we show also velocity profiles measured in the gap between the belts by LDV technique described in [8]. The profiles show a perfect simple shear flow pattern in the test section of the apparatus.

3. Results

The experiments were performed at several particle sizes and concentrations and at various shear rates, and are summarized in Table 1. A time print of the light intensity detected by a pixel of the CCD camera is shown in Fig. 8. The signal appears to be noisy, almost fully turbulent, and to have a broad spectrum of frequencies. We have taken such data and obtained ensemble averaged temporal and spatial spectra. Figs. 9–11 depict the collection of spectra that relate to the cases listed in the table. In Fig. 9, we show temporal spectra taken on the zero velocity line. Each curve is an ensemble average of spectra from 500 individual neighboring pixels. Since the intensity of the light recorded on the CCD camera (relating

<table>
<thead>
<tr>
<th>Particle diameter (μm)</th>
<th>Concentration (%)</th>
<th>Shear rate (s$^{-1}$)</th>
<th>Ca ($\times 10^{-3}$)</th>
<th>Slope x</th>
<th>Slope y</th>
</tr>
</thead>
<tbody>
<tr>
<td>655</td>
<td>40</td>
<td>2.0</td>
<td>5.7</td>
<td>$-2.8$</td>
<td>$-2.0$</td>
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<tr>
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<td>0.6</td>
<td>1.7</td>
<td>$-2.5$</td>
<td>$-1.9$</td>
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<tr>
<td>655</td>
<td>30</td>
<td>2.0</td>
<td>5.7</td>
<td>$-2.4$</td>
<td>$-1.7$</td>
</tr>
<tr>
<td>655</td>
<td>30</td>
<td>0.48</td>
<td>1.4</td>
<td>$-2.3$</td>
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<td>655</td>
<td>13</td>
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<td>50</td>
<td>2.0</td>
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<td>64</td>
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Fig. 8. A typical sample of raw data showing light intensity in arbitrary units vs. time.
amplitude of disturbance and local inclination of the surface) was arbitrary and uncontrolled, each curve was normalized with respect to the average intensity at the band of lowest frequencies. The points marked on the spectra were spread at constant frequency intervals and help to distinguish, in each plot, the three curves from each other. Note that the frequency of particle interactions suggested by Phillips et al. [2] is given by $G\phi$ where $G$ is the shear rate intensity and $\phi$ is the particle volume concentration, and it varies from about 0.06–1 in these runs.

Spatial power spectra were taken in two directions: in the direction of the flow ($x$-direction) and perpendicular to it ($y$-direction). They are depicted in Figs. 10 and 11, respectively. Each distribution is averaged over 20 individual uncorrelated different measured spectra. As in the presentation of the temporal spectra the curves were normalized with respect to the average intensity at the band of lower
wavenumbers. Note that the scale associated with the bigger particles is $1/\lambda = 15.3 \text{ cm}^{-1}$ while that associated with the smaller ones is $1/\lambda = 156 \text{ cm}^{-1}$. Note also that the maximum of the power lies at an approximate wavelength of $\lambda = 2 \text{ cm}$ and that the spacing in the gap between the belts is 2 cm as well.

4. Discussion

4.1. The experimental observations

We open the discussion with a reminder that, when particles are absent in the sheared test fluid, disturbances on the free surface were not defected and no spectra of corrugation could be collected.
Thus, the phenomenon is driven by the existence of particles and in sufficient concentration. When these conditions are fulfilled and a signal taken from a single pixel (e.g., Fig. 8) is transformed to the frequency domain, it is immediately apparent that the disturbance frequencies span a very wide range. Note that the spectra (Figs. 9–11) and the correlation functions (Figs. 12–14) are for the relative illumination intensity at the surface that is associated with the local inclination of the surface, hence, they are qualitatively related to the wave amplitude.

The temporal power spectra depicted in Fig. 9 reveal continuous spectra, monotonically decreasing with the increase of frequencies, with no apparent concentration of power at a particular or a specific frequency. There seems to be a dependence on the particle concentration; however, it is not dramatic at this range. It is more apparent for the smaller particle size. No significant dependence on particle size is
detected. The spectra of the two sizes have similar shape and similar frequency distribution for a given flow rate. The dependence on shear intensity is more significant. Less power is distributed in the higher frequencies as the shear intensity decreases. This may be related to the amount of inertia associated with the macroscopic bulk flow. Note that the Reynolds number of the flow, for the high shear rate ($G = 2 \text{s}^{-1}$) is of $O(1)$ and, hence, the inertia at the surface cannot be entirely neglected.

The spatial PSD, in both $x$- and $y$-directions, possess a form typical to a turbulent system. They show an energy transfer cascade along wavelengths and decay rates, visible as the slopes, that depend on the system parameters. The slopes are given in Table 1. A slight dependence on particle concentration and on particle size is evident as the slopes increase monotonically with concentration and size. However, changes in the shear rate do not have a significant influence. It suggests that the distribution of power along
Fig. 13. Normalized spatial auto-correlation functions in the direction of the flow (x-direction). For captions of (a–d) see Fig. 12.

The cascade of wavenumbers is influenced by the effective viscosity and effective diffusivity, and that higher concentrations enhance the dissipation of power at the low wavelength region. Note that there are two main differences between the x-direction PSD depicted in Fig. 10 and the y-direction PSD shown in Fig. 11. The slopes in the y-direction are lower and, thus, more power is spread, relatively, towards shorter waves. The other difference is the intensity of the fluctuations. They are much weaker in the y-direction as if the fluctuations are inhibited by an external influence that also directs them to be distributed over a wider range of waves. Indeed the main difference between the two types of spectra is that the y-direction is confined between the two belts while the x-direction is unbounded.

It is interesting to note that no special information is revealed in the spectra that relates to the particle scale. The energy input scale into the system is well defined, as it is the steel belts that supply the energy and these are separated 2 cm apart. However, the disturbances are excited at the particle scale and are dispersed up and down the waves scale ladder. Thus, we see an inverse cascade in the transfer of the
disturbance energy from an intermediate scale (particle) to the longer waves and, simultaneously, a more common cascade of transfer toward higher wavenumbers.

Complementing the temporal and spatial spectra we have used the same data to obtain auto-correlation functions. These are shown in Figs. 12–14, respectively. It is evident from Fig. 12 that the temporal correlations decay rate increases with concentration, \( \phi \), between 0.13 and 0.40, and that it also increases with the shear rate. These are the two parameters that promote the turbulent disorder. On the other hand, no special dependence of the decay rate on particle size is observed. Furthermore, for the smaller particle size there is a maximum around \( \phi = 0.4 \) and the correlation for \( \phi = 0.5 \) declines less strongly. This is also apparent in the turbulence intensity spread shown in Fig. 9c and d. This maximum agrees well with the report of a maximum in the intensity of shear-induced particle migration at \( \phi \) between 0.4 and 0.45 (see, e.g. Shauly et al. [9]).
The spatial correlations in the $x$-direction, depicted in Fig. 13, suggest that for the smaller particles, no special dependence exists between the decay distance and the particle concentration. At the higher particle size we find that the decay distance increases with the particle concentration. In most cases, a negative correlation is evident, which indicates a small periodic contribution to the signal. Note that the wavelength associated to a negative correlation is twice as large as the location of the minimum, at the position where the subsequent maximum could be expected, were it not already damped out. For the lower particle concentrations, the minimum of this negative ‘valley’ is found at a distance equivalent to the large particle size. However, for $\phi = 0.4$ the scale is several times bigger. It is possible that this larger scale relates to the formation and migration of clusters of the larger particles that agglomerate at the higher concentration. No direct evidence to this possibility was detected. Negative correlations in the $y$-direction spectra, when detected, appear to be much more confined (Fig. 11). This is a manifestation of the decay rate of power density with wavenumber being much smaller in $y$-direction than in $x$-direction.

We also recall here that the curvature of the correlation function at the origin is related to the size of the smallest scale present in the signal. In the correlation functions for the large particles, Figs. 13 and 14, this scale is much smaller than the particle size. Such small scales are evident in Fig. 5, where there are several quite sharply contoured wave crests present, which probably introduce these small scales into the system. However, as can be also seen in the power spectra (e.g. Fig. 10) the spectra of the smaller particles span over a wider range than the corresponding spectra of the larger particles, and this is also expressed in the initial slopes in Fig. 13.

4.2. A model for free surface corrugation

The experimental apparatus described in Section 2, was designed to establish a two-dimensional flow field with a uniform simple shear flow and, hence, a uniform shear intensity everywhere. This was accomplished and validated by measuring velocity profiles in a pure homogeneous viscous fluid. The suspension was introduced into the system with a uniform particle volume concentration. Thus, in the absence of shear intensity and particle concentration gradients, no net flux of particles is expected and only self-diffusion of particles was anticipated. However, at the free surface, the suspension concentration experiences a sharp normal gradient where it suffers a jump from the bulk concentration, $\phi$, to zero across a length scale of the order of the particle size. Particles located near the interface experience interactions with other particles located in the suspension side while they have fewer or no interactions on the other side. The anticipated result is the existence of shear-induced particle flux toward the interface. This flux is balanced by the surface tension restoring force. The balance between these two mechanisms and their relative intensity can serve to provide a measure to the roughness witnessed on the surface.

In the absence of shear intensity gradients, the shear-induced flux of particles to the surface is given by

$$q_{\text{out}} = -D \frac{\partial \phi}{\partial z}$$  \hspace{1cm} (1)

where $z$ denotes a co-ordinate normal to the suspension surface pointing out-ward and

$$D = \hat{D}(\phi) G a^2$$  \hspace{1cm} (2)

with $G$ and $a$ being the constant shear intensity and particle radius, respectively and $\hat{D}(\phi)$ is a dimensionless function of the particle concentration. Across the thin layer, in which the concentration shrinks...
from $\phi$ to 0, the normal gradient can be approximated by $\frac{\partial \phi}{\partial z} = -\frac{\phi}{2a}$. Next we estimate the force that prevents the particles from separating themselves from the surface. When a particle penetrates the interface, and is sticking out a height $l$ beyond the undisturbed surface level, the roughness can be characterized by the ratio $l/a$, as depicted in Fig. 15. The capillary force acting to push the particle back into the suspension is estimated by

$$F = \int_{\text{exposed surface}} \gamma (\nabla \cdot \mathbf{n}) \, dS \approx 2\pi \gamma \left( 2 - \frac{l}{a} \right) t$$

(3)

where it is assumed that the particle remains coated with a film of fluid and that it does not stick out of the surface more than its radius, i.e. $l/a < 1$. The quantity $\pi d^2 = \pi (2al - l^2)$ is merely the projection of the exposed surface onto the undisturbed one. Hence, with this force, the flux of particles pushed by interfacial tension back into the suspension is given by

$$q_{\text{in}} = \frac{F\phi}{6\pi a\mu_{\text{eff}}}, \quad \mu_{\text{eff}} = \lambda(\phi)\mu$$

(4)
where $\mu_{\text{eff}}$ and $\mu$ are the effective viscosity of the suspension and the viscosity of the pure fluid, respectively. Upon equating these fluxes we find that

$$\frac{l^2}{a^2} \approx 1 - \sqrt{1 - \frac{3}{2} \lambda(\phi)\hat{D}(\phi)Ca}, \quad \text{Ca} = \frac{\mu Ga}{\gamma}$$

where $\text{Ca}$ is the capillary number. Note that when $(3/2)\lambda(\phi)\hat{D}(\phi)Ca \ll 1$, the roughness can be approximated by $l/a = \left(\frac{3}{4}\lambda(\phi)\hat{D}(\phi)Ca\right)^{1/2}$.

The above prediction suggests that corrugation will be small in the following cases: dilute suspensions; small particles; large surface tension; low shear rate and low viscosity. However, in the opposite cases, i.e. when the suspension is concentrated and when $\text{Ca}$ is not negligible the roughness of the interface caused by the particles surface shear-induced dynamics can be quite significant. For example, at $\text{Ca} = 0.01$ for any suspension concentration $\phi$ for which $\lambda(\phi)\hat{D}(\phi) \gtrsim O(10)$ we should expect a non-negligible roughness of more than a few percents. Fig. 16 shows the expected surface roughness as function of the particle concentration for various $\text{Ca}$. The curves were calculated using the expressions for $\lambda(\phi)$ and $\hat{D}(\phi)$ originally proposed by Leighton and Acrivos [1]. The shaded area indicates the range of parameters in our experiments, and the expected roughness there.

4.3. Possible effect on surface stability

There is evidence that a free interface in a sheared suspension does not always maintain a homogeneous unperturbed state. Tirumkudulu et al. [5] report observations in suspensions of mono-disperse neutrally buoyant particles. When sheared in a partially filled horizontal Couette device, the suspension separated itself into alternating regions of high and low particle concentration along the length of the tube. The phenomenon repeated itself when the experiment was performed in a following study [6] inside a single
partially filled cylinder. Note that in these studies the direction of the velocity, the plane of shear and the normal to the unperturbed surface are all orthogonal to each other. Thus, the intensity of shear in these experiments is not uniform. It has a maximum near the rotating cylinder while at the free surface the shear intensity is diminished and so is the shear-induced particle diffusivity. It is, therefore, expected that any shear-induced surface roughness should be minor if detectable at all. In our experiments, the plane of shear and the free surface are oriented in the same direction. The shear intensity is uniform throughout the flow field and particularly at the free surface. Hence, the pronounced corrugated surface.

The dynamics of the particles at the interface increases the size of exposed surface significantly and influences the amount of free energy there. It is tempting to define effective surface properties in a manner similar to the definition of other effective bulk properties of suspensions such as density or viscosity (see, e.g. Batchelor [10]).

The local stress balance at the surface is given by
\[ \Delta f = \gamma (\nabla \cdot n)n \] \hspace{1cm} (6)

where \( \Delta f \) is the difference of surface tractions (outer − inner). Following Batchelor [10], we consider a microscopic length scale in which there is a statistically significant number of realizations of the corrugation causing events, while it is small compared to the macroscopic scale, \( L \), of the surface. Taking an ensemble average we obtain
\[ \langle \Delta f \rangle = \gamma \langle (\nabla \cdot n)n \rangle \] \hspace{1cm} (7)

where \( H \) denotes the local curvature. We express the local curvature as a combination of the unperturbed curvature and the local deviation from it, \( H = \bar{H} + H' \). Similarly for the unit vector normal to the surface we have \( n = \bar{n} + n' \). We assume for simplicity that the perturbations of the surface, induced by the particle shear-induced migration, do not interact with each other. In this case, we can approximate the ensemble average in the RHS of Eq. (7) by
\[ \langle Hn \rangle = \frac{1}{2} \sum_S \int_S (\nabla \cdot n)n dS \] \hspace{1cm} (8)

Here \( S \) is the surface area on which the average is performed and the summation is on all particles at this surface. With reference to Fig. 15, taking into account the symmetries around each particle and the approximation \( H(L) = 2/L \), a first order estimate of Eq. (8) to \( O(\phi) \) is
\[ \langle Hn \rangle = \frac{1}{2} \sum_S \int_S \left\{ \frac{2}{L} + \left[ \left( \frac{2}{a} - \frac{2}{L} \right) - \frac{4a}{\pi} \left( \frac{1}{a} - \frac{b}{\alpha} \right) \right] \phi \right\} \] \hspace{1cm} (9)

The sign of the \( O(\phi) \) correction in Eq. (9) depends on the relative sizes of the various radii of curvature, \( L, a, d \) and \( b \). While \( a \) is, naturally, much smaller than \( L \), the relative magnitude of \( d \) and \( b \) depends on the rate of approach of the particle to the surface. Geller et al. [11] studied the deformation of a free interface by the approach of a small spherical particle. From their results we can conclude that \( d \) and \( b \) are relatively similar in size and, therefore, to this degree of approximation we can neglect the last term in Eq. (9). Furthermore, since a dimensional analysis of a particle balance yields that the ratio \( L^2/\alpha^2 \) is the Peclet number (typically being very high) we obtain that
\[ \gamma \langle Hn \rangle = \gamma \bar{H}(1 + \phi \sqrt{Pe}) \bar{n} \] \hspace{1cm} (10)

Eq. (10) suggests that in most cases with the above approximations, the effective surface tension increases with the particles interfacial concentration. Thus, we can anticipate the onset of local Marangoni type flows
because of the tendency of the surface to flow toward high tension regions, and this surface induced motion will increase the local particle concentration even further. The local change of particle concentration will also render local variations of effective viscosity, surface mobility and particle migration. This suggests a mechanism in which such positive feedback can be a source for interfacial instabilities that can drive processes such as surface particle segregation or transition to free surface turbulence. We expect that future studies will help to assess the existence and significance of this effect.

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References